TO THE INVESTIGATION OF OSCILLATIONS DESCRIBING THE GENERALIZED DIFFERENTIAL RAYLEIGH EQUATION

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Self-oscillation is a common variant of cyclic change in time of the motion parameters of different systems, caused by a non-oscillating energy source. They deal with acoustics, electrical and radio engineering, mechanics, metalworking, agricultural production, and so on. Therefore, it is important to know the laws of self-oscillations, in the modeling of which the Rayleigh equation plays an important role.

Self-oscillation is a type of nonlinear oscillation. Therefore, in nonlinear dynamics, as a rule, due attention is paid to self-oscillations. This is observed for a long time [1, 2]. In addition, a special edition [3] and others are devoted to self-oscillation. In recent years, the theory developed has been used to solve various engineering problems. Thus, the simplest version of the theory is known, when the instantaneous change in the value of the coefficient of friction is accepted, was used in [4], in the study of self-oscillations of the scraper conveyor, and also considered in [5, 6] on a more general basis.

In terms of mechanics, displacement is described by the equation:

$$m\ddot{x} - k_1\dot{x} + k_2\left|\dot{x}\right|^{\nu}\operatorname{sign}\left(\dot{x}\right) + c\,x = 0\,, \tag{1}$$

in which $m-$ oscillator mass; $k_1 > 0, k_2 > 0-$ coefficients of resistance; $\nu \geq 0-$ nonlinearity index; $c-$ coefficient of elasticity.

We use the results of [7], which considered the free oscillations of the oscillator described by the differential equation:

$$m\ddot{x} + k_1\dot{x} + k_2\left|\dot{x}\right|^{\nu}\operatorname{sign}\left(\dot{x}\right) + c\,x = 0\,. \tag{2}$$

Equation (2) differs from (1) only by the sign before k_1. Therefore, we will further use the results of [11]. According to it:

$$x(t) \approx a(t)\cos(\omega t); \tag{3}$$

$$a(t)=\begin{cases}\left[b+\left(a_0^{1-\nu}-b\right)e^{\lambda_1(\nu-1)t}\right]^{\frac{1}{1-\nu}} & \nu\neq 1\\ a_0=a_0\exp\left(\dfrac{k_1-k_2}{2m}t\right) & \nu=1;\end{cases}\quad \text{when} \qquad (4)$$

$\omega=\sqrt{c/m}$; $b=\dfrac{\lambda_2}{\lambda_1}$; $\lambda_1=\dfrac{k_1}{2m}$; $\lambda_2=\dfrac{k_2\omega^{\nu+1}}{\pi c}\dfrac{\Gamma\left(\frac{\nu+2}{2}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\frac{\nu+3}{2}\right)}$; $a_0=a(0)-$ the initial deviation of the system from the equilibrium position; $\Gamma(z)-$ gamma function.

Depending on the nonlinearity index ν, we have three cases.

Under the condition $\nu>1$ equals (1) - (4) describe quasilinear self-oscillations that have a steady-state amplitude that does not depend on the initial deviation. In the case when $\nu=1$ we have: $k_1<k_2$ free damped oscillations, where the decrease in amplitudes occurs according to the law of geometric progression, and the oscillating process is not limited in time; if $k_1>k_2$, then there is a oscillation oscillation, subject to the exponential law; if $k_1=k_2$ – free undamped oscillations with constant amplitude.

The study showed that the generalized Rayleigh equation, depending on the value of the nonlinearity in the expression of the resistance force, can describe both quasilinear self-oscillations with a steady state independent of the initial conditions and free oscillations with a limited number of cycles before their termination.

References

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