MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE

State Biotechnological University

Faculty of Mechatronics and Engineering

Department of Physics and Mathematics

EDUCATIONAL AND METHODOLOGICAL LITERATURE

SUMMARY OF LECTURES

FROM EDUCATIONAL DISCIPLINE

PHYSICS

PART I

for acquirers

level of higher education first (bachelor) full-time (part-time) study in specialty 133 "Industrial mechanical engineering"

Kharkiv

2023

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SUMMARY OF LECTURES
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PHYSICS
PART I

for acquirers

level of higher education first (bachelor) full-time (correspondence)

forms of education in specialty 133 "Industrial mechanical engineering".

Approved

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Educational and methodical literature. Synopsis of lectures on the academic discipline "PHYSICS" Part I, for students of the first (bachelor's) full-time and correspondence form of higher education in specialty 133 "Industrial mechanical engineering". / DBTU; editor: Pak A.O., Sinyaeva O.V., Krekot M.M. Kharkiv, 2023. 57 p.

Educational and methodical literature. The synopsis of lectures on the academic discipline "PHYSICS" was developed in accordance with the initial program. The history of the development of physics, the scientific path of outstanding scientists, the main laws and formulas from the discipline of physics, examples of experimental research are given, the physical foundations of the mechanics of translational motion, the mechanics of rotational motion, molecular physics and thermodynamics are discussed.

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Responsible for the issue: A.O. Pak, Ph.D.

Зміст

Lecture 1
INTRODUCTORY LECTURE
Plan

1. The subject of physics, the goal of the discipline.
2. Connection of physics with other sciences and practice. Methods of physical cognition.
3. The main stages of the development of physics.
4. Physics as the basis of technical training of engineering personnel. The role of physics in agriculture.
5. Human ecology from the point of view of physics.

1. The subject of physics, the goal of the discipline

The science of nature received the name "Naturwissenschaft". The international term "natural philosophy" (philosophy of nature) comes from this name.

Initially, all knowledge about nature belonged to the sphere of interests of physics (or physiology). It is no accident that Aristotle (4th century BC) called his predecessors "physicists" or "physiologists" (the ancient Greek word "physis" is very close in meaning to the Slavic word "nature"). **It is physics that is the basis of all natural sciences.**

The subject of physics is: the study of the most general forms of movement of matter and their mutual transformations.

The goal of the discipline is to provide students with the basics of theoretical training; formation of scientific thinking and dialectical-materialistic outlook; assimilation of basic physical laws by students; acquisition by students of techniques and skills for solving standard and non-standard problems from various sections of physics; formation of subject, branch and key competences by means of physics as an educational subject, development of the ability to independently solve various scientific and technical tasks at the modern level of science, technology and technology.

As a result of studying physics, the student should

be able to:

- use measuring devices, physical equipment and computer equipment;
- conduct physical experiments;
- process the obtained experimental data and evaluate their reliability;
- build appropriate schedules;
- to assess the degree of negative impact of this or that technology process on the environment, to predict and eliminate it in advance;
- apply acquired theoretical and practical knowledge to explain physical processes and phenomena that occur during the operation of modern mechanisms, devices and equipment.

2. Connection of physics with other sciences and practice. Methods of physical cognition.

The Greek word "physis" in translation means "nature", so the science of nature began to be called physics.

Physics is a fundamental science, because all other natural sciences (chemistry, geology, biology, etc.) deal with certain types of material systems that obey the laws of physics. For example, the properties of chemicals are determined by the properties of the molecules and atoms that make them up, and these properties are studied in such branches of physics as quantum mechanics, thermodynamics, and/or electricity (electromagnetism).

Physics is closely related to mathematics. Physical theories, as a rule, are built on the basis of a certain mathematical apparatus, and this apparatus is often much more complicated in comparison with other natural sciences.

The role of physics and all natural sciences in human life is difficult to overestimate. It is the basis of life support - physiological, technical, energetic. This is the theoretical basis of industry and agriculture, all technologies, various types of production, including the production of energy, food, clothing, etc. Natural science is the most important element of human culture, one of the most important indicators of the development of civilization.

Physics uses various techniques and methods of cognition (research): observation, measurement, experiment, comparison, induction, deduction, analysis and synthesis, abstraction and generalization, scientific hypothesis, modeling, system analysis, imaginary experiment, etc. The most important feature of the natural sciences, unlike the humanities, is their experimental nature.

The path to knowledge in physics can be imagined as follows: ***observation - a hypothesis to explain the observation - an experiment to test the hypothesis - development of a theory (if the hypothesis is confirmed) - verification of the consequences arising from the theory***. It should be noted that theory is the main form of knowledge, its accumulator. According to the words L. Boltzman, "there is nothing more practical than a good theory." This, of course, does not deny the role of practice as a criterion of truth. Theory and experiment, as the two most important methods of knowledge, are in a dialectical unity, the violation of which leads to the fact that theory becomes an objectless scheme, and experiment becomes blind.

3. The main stages of the development of physics

The development of physics is divided into three main stages.

The first stage. Observation of physical phenomena took place since ancient times. At that time, the process of knowledge accumulation was not yet differentiated; physical, geometric and astronomical ideas developed together. The economic need to separate land plots and measure time led to the development of space and time measurements even in the past - in Egypt, China, Babylonia.

The systematic accumulation of facts and their explanation and generalization took place with particular intensity in the era of Greek culture (6th century BC - 2nd century AD). In this era, the initial ideas about the atomic structure of matter were born (Democritus, Epicurus, Lucretius), the geocentric system of the world was created (Ptolemy), some simple laws of statics were established: the rules of the lever, the center of gravity (Archimedes), the first results of applied optics were obtained (made mirrors, Euclid discovered the laws of geometric optics), discovered the simplest principles of hydrostatics (Archimedes' law). The simplest phenomena of magnetism and electricity have been known since ancient times.

Aristotle's teachings summarized the knowledge of the previous time. One of the main books of Aristotle is called "Physics". Despite some incorrect statements, Aristotle's physics has remained the basis of knowledge about nature throughout the ages.

The second stage is classical physics of the 16th-19th centuries.

Almost one and a half thousand years separates the geocentric system from the rather perfect heliocentric system of the Polish mathematician and astronomer Nicolaus Copernicus (1473-1543). The top of the heliocentric system can be considered the laws of planetary motion discovered by the German astronomer Johann Kepler (1571-1630), one of the creators of modern astronomy. The astronomical discoveries of Galileo Galilei and his physical experiments, as well as the general dynamic laws of mechanics together with the universal law of universal gravitation, formulated by Isaac Newton in 1687 in the scientific treatise "Mathematical Principles of Natural Philosophy", marked the beginning of the classical stage of the development of physics.

Not only classical Newtonian mechanics developed at a rapid pace. The stage of classical physics is also characterized by great achievements in other fields of physics: thermodynamics, molecular physics, optics, electricity, magnetism, etc. Gas laws were established. Equations of the kinetic theory of gases are proposed. The principle of uniform distribution of energy by degrees of freedom, the first and second laws of thermodynamics is formulated. Open laws of Coulomb, Ohm and electromagnetic induction. The phenomena of interference, diffraction and polarization of light received a wave interpretation. The laws of absorption and scattering of light are established.

A special place is occupied by the electromagnetic theory developed by the outstanding English physicist James Clerk Maxwell. Maxwell is not only the creator of classical electrodynamics, but also one of the founders of statistical physics. Developing the ideas of Michael Faraday (1791-1867), he created the theory of the electromagnetic field (Maxwell's equation), which not only explained many electromagnetic phenomena known at that time, but also predicted the electromagnetic nature of light.

A characteristic feature of the third stage of the development of physics - the modern stage is that, along with the classical ones, quantum ideas are widely introduced, on the basis of which many micro-processes that take place within the atom, nucleus and elementary particles are explained, and in connection with which new fields have arisen modern physics: quantum electrodynamics, quantum solid state theory, quantum optics and many others.

In 1900, Max Planck put forward the quantum hypothesis according to which atomic oscillators do not emit energy continuously, but in certain portions - quanta, and the energy of a quantum is proportional to the frequency of oscillation. In 1905, Albert Einstein applied Planck's idea to successfully explain experiments on the photoelectric effect. In 1911, Ernest Rutherford proposed the planetary theory of the atom, and in 1913, Niels Bohr constructed a model of the atom in which he postulated the quantum nature of electron motion. Thanks to the works of Werner Heisenberg, Erwin Schrödinger, Wolfgang Pauli, Paul Dirac and many others, quantum mechanics found its exact mathematical formulation, confirmed by numerous experiments.

With the discovery of radioactivity by Henri Becquerel, the development of nuclear physics began, which led to the emergence of new sources of energy: atomic energy and nuclear fusion energy.

The leading edge of physics moved into the area of research of the most fundamental laws, setting itself the goal of creating a theory that would explain the universe by combining theories of fundamental interactions. On this path, physics achieved partial success in the form of the theory of the electroweak interaction and the theory of quarks generalized in the so-called standard model. However, the quantum theory of gravity has not yet been constructed.

4. Physics as the basis of technical training of engineering personnel. The role of physics in agriculture

Acceleration of scientific and technological progress provides the state with enormous opportunities for the development of productive forces, improvement of the human personality, and the construction of harmonious relations with nature.

The most important sections, on which the great contribution of physics to the development of agriculture is based, are mechanics, molecular physics, electrodynamics and nuclear physics.

In recent years, there has been a rapid development of the field of technology based on the physics of semiconductors - optoelectronics. First of all, this is manifested in the rapid revolutionary improvement of LEDs - solid-state semiconductor light sources. If 15-20 years ago, light-emitting diodes were known to most people as display devices, in the laboratories of leading scientific centers, new technologies for the production of semiconductors were developed, which made it possible to change the world of artificial lighting for the better, replacing it with classical light sources (incandescent lamps, fluorescent lamps, etc.) more efficient and technological - LED. Until recently, the lighting of premises for poultry and animals, as well as other places related to the processing of poultry and livestock products, was carried out by "classic" light sources, such as incandescent lamps and fluorescent lamps. Theoretical studies and more than five years of practical experience allow us to conclude that LED systems used in agriculture reduce electricity consumption for lighting enclosures for poultry and animals by 8-10 times compared to incandescent lamps and in 1.8-2.2 times compared to fluorescent lamps.

The direct application of ionizing radiation as a process of radiation-biological technology (RBT) is relevant for: - sterilization, preservation, increase in shelf life and disinfection of food products and fodder, raw materials of animal origin (wool, leather, fur, etc.). Plants for irradiating vegetables and fruits in order to protect them from rotting and mold have been used in agriculture.

The use of new, most effective physical principles of action in various production processes is of great importance. An example can be the improvement of water purification methods due to the transition from evaporative systems to membrane technologies. Cost effectiveness when solving tasks of a similar class increases 8-10 times.

The main directions of use of production waste and substances captured by treatment plants are their return to production as raw materials and semi-products, use as a finished product and fuel; in agriculture - as plant growth regulators and for soil neutralization; in the production of building materials - as raw materials. Thus, the problem of rational use of secondary material resources (and on the basis of this reduction in the need for primary, including natural) combines the interests of nature protection with increasing the economic efficiency of production.

To date, methods of complex energy-technical use of low-grade solid fuel have been developed, from which, with the help of thermal decomposition, high-quality solid, liquid and gaseous fuel, as well as raw materials for the chemical industry and the production of building materials, are obtained. The ash residue is used in agriculture.

The Ministry of Regional Development, Construction and Housing and Communal Services published the report "Development of renewable energy sources in Ukraine", prepared as part of the "Secretariat and Expert Hub on Energy Efficiency" project, implemented by the United Nations Development Program in Ukraine with the support of the Government of the Slovak Republic and with the assistance of the Ministry of Regional Development .

The report analyzed the trends in the development of renewable energy sources, prepared an overview of the current state of the energy sector and the potential of renewable energy in Ukraine, considered financing mechanisms, key obstacles and recommendations for authorities to eliminate them.

Over the past decade, the world has seen a steady trend towards the development of renewable energy sources that are gradually replacing traditional generation. Alternative energy sources - renewable energy sources, which include solar, wind, geothermal, wave and tidal energy, hydropower, biomass energy, gas from organic waste, gas from sewage treatment plants, biogas, and secondary energy resources, which include blast furnace and coke gases, methane gas, degassing of coal deposits, conversion of waste energy potential of technological processes In 2015, global investments in renewable energy sources (RES) amounted to a record 349 billion dollars. In Ukraine, there has been an increase in installed renewable energy capacity over the past 4 years, but the difficult economic situation did not allow achieving the goals adopted in the National Action Plan for Renewable Energy. At the end of 2016, 1,117 MW of RES capacity was installed, producing about 1% of electricity in Ukraine. The largest share is occupied by wind and solar power plants (925 GW*h and 492 GW*h of generated electricity, respectively).

According to experts' estimates, the economically feasible potential of RES implementation in Ukraine as of 2030 is estimated at 16-22 GW, compared to 1.1 GW actually installed at the end of 2016. The potential for the introduction of RES in heat energy is even greater, in this segment RES can completely replace traditional energy sources by 2030. According to IRENA estimates, in 2030 about 57 million Gcal of thermal energy can be produced from RES, of which a significant share (32.7 million Gcal) is biomass energy. Fulfillment of this forecast will save about 7 billion cubic meters. of natural gas every year. The development and use of alternative and renewable energy sources (wind and solar energy, biofuel, etc.) is an important factor for strengthening energy security and reducing the negative man-made impact on the natural environment. The importance of the development of alternative energy is obvious, because it plays a decisive role in reducing greenhouse emissions, reducing the negative impact on the environment, increasing the security of energy supply, and helping to reduce dependence on energy imports. It goes without saying that all RES technologies and principles are based on the laws and phenomena of physics.

Advances in space technology are also promising. Weather forecasts, provided through a network of satellites and other means of communication, help people make decisions about when to sow, water, fertilize, and harvest. Remote sensing and satellite imaging can ensure optimal use of

Earth's resources, allowing monitoring and assessment of long-term trends in climate change, marine pollution, soil erosion rates, and vegetation cover.

5. Human ecology from the point of view of physics.

Environmental problems are considered to be a number of factors that cause the degradation of the natural environment. In most cases, they are the result of human activity. In particular, the environmental problems of the Earth arose due to the increased development of technology and industry, as problems began to appear that are directly related to the violation of uniform conditions inherent in the ecological environment, which are very difficult to compensate for today. One of the most destructive circumstances of human activity is pollution. It manifests itself as an increased level of smog, the emergence of dead lakes, technical water in which there are harmful elements, as a result of which they become unfit for consumption. Also, pollution provokes the extinction of some species of inhabitants of the animal world. From this we can conclude that a person, creating favorable conditions for himself, has a detrimental effect on the environment. In connection with this, modern humanity begins to pay more attention to this issue. The result of this is the work of scientists, which examines environmental problems in detail and their solution by searching for alternative options. That is why it is so important for future agribusiness specialists to know the main environmental problems of our time, which have a physical nature and ways to solve them. As an example, we can take a local environmental problem caused by a factory that does not undergo appropriate treatment of industrial effluents before discharging them into the river. Such human actions have very deplorable consequences, as fish die and the health of the entire society suffers. If we consider the regional level, then the Chernobyl zone stands out, on the territory of which there is contaminated soil. It is endowed with radioactive properties and threatens all biological organisms that are in this area. All these problems have a physical origin to one degree or another. Global environmental problems need no less attention. They stand out on a huge scale and directly affect a whole range of ecological systems. This is where they differ from regional and local issues of ecology. The main factors of environmental problems are the following natural conditions:

- warming of climatic conditions;
- formation of ozone holes.
- large amount of fuel burning;
- accumulation of carbon dioxide in the atmosphere.

These factors caused a significant violation of heat transfer and slowing down of air cooling.

An equally important problem for humanity is the ozone hole, which arose as a result of technological progress. As you know, the birth of life on Earth was facilitated by the formation of the protective ozone layer, which serves as a reliable protection for organisms against strong ultraviolet radiation. However, in the 20th century, scientists discovered that Antarctica contains an extremely small amount of ozone.

It is in this position to this day. The damaged area of Antarctica is compared to the area of North America. Active launches of satellites, rockets and aircraft are called the main cause of its occurrence.

Acid rain is caused by the operation of power plants. Such phenomena provoke other ecological problems of the Earth, such as the loss of forests. So, about 70% of forests in the Czech Republic and Slovakia were destroyed by these rains, a similar situation can be observed in Germany and Great Britain, where 60% of forest areas were destroyed.

Such desertification is a global problem. Its essence lies in the deterioration of the soil, as a result of which large areas have become unsuitable for use in agriculture. Annually, 20 billion tons of carbon dioxide enter the atmosphere as a result of burning fuel. Only when coal and fuel oil are used, more than 150 million tons of sulfur gas are released. About 160 km3 of industrial effluents are discharged into rivers every year. During the same time interval, more than 500 million tons of mineral fertilizers and approximately 3 million tons of toxic chemicals are introduced into the soil, a third of which is washed into the waters of the land and the ocean.

Dangerous phenomena are being observed that can radically change the appearance of the planet, threaten the existence of many species of plants and animals, and represent a danger to the human race. Annually, approximately 6 million hectares of productive land turns into desert. In three decades, the area subject to desertification will be roughly the size of Saudi Arabia. More than 11 million hectares of forest are destroyed every year, and in three decades the area of lost forests will be roughly equal to the area of India. A large part of the area where forests used to grow is turning into low-quality agricultural land that cannot feed the people who live on these lands. As a result of the burning of mineral fuel, carbon dioxide is released into the atmosphere, which is the cause of the gradual warming of the global climate. As a result of such a "greenhouse effect", average global temperatures may rise in the 21st century. so much so that areas of agricultural production will change, seas will break their banks and flood coastal cities, and the economy will suffer serious losses.

Other gases of industrial origin are able to damage the protective ozone layer of the planet, as a result of which the number of human and animal cancers will increase dramatically.

The ozone screen (ozonosphere), located at an altitude of 10-50 km, is the atmospheric zone with the maximum amount of ozone. The ozone layer owes its existence to the activity of photosynthetic plants and the action of ultraviolet rays on oxygen, it protects all life on Earth from the harmful effects of these rays. In recent years, scientists have been concerned that the thickness of the ozone layer is gradually decreasing. On the basis of the wide use of the latest achievements of scientific and technical progress, there is an opportunity to create a new progressive technology, hardware design corresponding to it, on which productions are based, which in their essence become ecologically clean, do not harm the environment. The simultaneous solution of economic, technical, organizational and environmental problems of the development of social production at lower costs is realistic.

Lecture 2
CONTENT MODULE 1. MECHANICS

Topic 1
KINEMATICS and DYNAMICS OF PROGRESSIVE MOVEMENT. FORCES IN MECHANICS

Kinematics of translational movement

Plan

1. Mechanical movement. Reference system. Kinematic equations of motion. The path and movement of a material point.
2. Average and instantaneous speeds.
3. Acceleration. Total, normal, tangential acceleration. Their directions and the relationship between them in vector and scalar forms.
4. Traffic classification.

1. Mechanical movement. Reference system. Trajectory, path and movement

Mechanics is the study of the simplest forms of motion of matter, which consists in the movement of one body relative to another or the relative movement of body parts over time.

Mechanics is divided into kinematics, dynamics and statics.

Kinematics studies body movement regardless of the causes that cause it.

Dynamics studies the laws of motion and the causes that determine it.

Statics studies the laws of equilibrium of bodies.

Let's get acquainted with some concepts that will be used in the future.

A mechanical system is a set of bodies considered in a particular problem.

A material point is a body whose dimensions can be neglected compared to the distances to other bodies considered in this problem. For example, when studying the movement of the Earth around the Sun, it can be considered a material point. An arbitrary system of bodies or a macroscopic body can be considered as ***a system of material points***.

An absolutely rigid body is a body whose deformations can be neglected in this problem.

Body movement always occurs in time and space. The absolute movement of the body, out of relation to other bodies, is meaningless. Movement is always relative, just as the very concept of space is relative.

A body (set of bodies), which is conventionally considered stationary and relative to which the movement of other bodies is considered, is called a reference body.

A frame of reference, which is given a coordinate system and a clock, is a frame of reference.

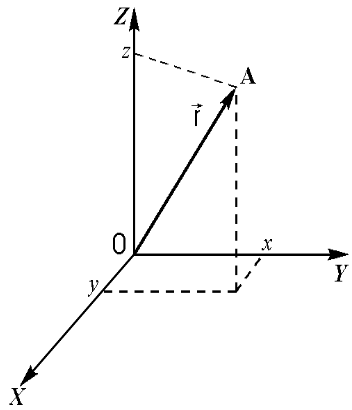


Fig. 1.1

The Cartesian coordinate system is most often used to describe the position of a point in space. In this system, the position of a material point is completely determined by specifying its three coordinates *x, y, z* or by the radius vector $\vec{r}$ drawn from the origin of coordinates to this point A (Fig. 1.1):

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} ,$$

where are $\vec{i}$, $\vec{j}$, $\vec{k}$ coordinates, unit vectors of the coordinate axes.

When a point moves, its coordinates and radius vector change over time. For the task of the law of motion, it is necessary to specify the dependence on time or coordinates, or the radius-vector:

$$x = x(t); \quad y = y(t); \quad z = z(t). \tag{1.1a}$$

$$\vec{r} = \vec{r}(t). \tag{1.1б}$$

Equations 1.1a and 1.1b are kinematic equations of motion.

A trajectory is a line "drawn" by a point in the process of movement. Depending on the shape of the trajectory, the movement can be rectilinear or curvilinear. In translational motion, any straight line connected to the body remains parallel to itself. At the same time, all body points describe similar trajectories (Fig. 1.2). That is why in the future, when considering the translational motion of a rigid body, there is no need to describe the motion of all its points - it is enough to consider the motion of any one point, for example, as is often done - the center of mass of the body.

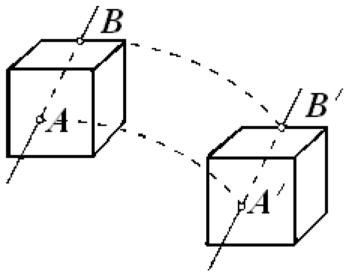


Fig. 1.2

Let a material point, moving along the trajectory (Fig. 1.3), at a certain moment in time take position *A*. We denote its radius-vector drawn from the origin of coordinates as $\vec{r}_0$. After a certain period of time Δt, the point will take position *B* with radius-vector $\vec{r}$.

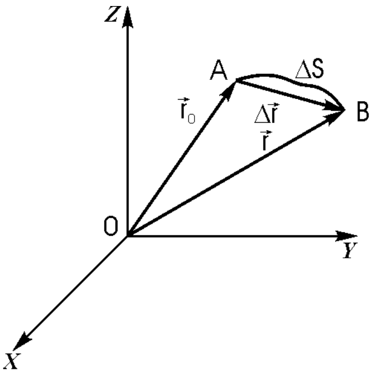


Fig. 1.3

The vector $\Delta\vec{r} = \vec{r} - \vec{r}_0$ drawn from the initial to the final position of the moving point is called its displacement (Fig. 1.3). Otherwise, the displacement is equal to the increase of the radius-vector $\vec{r}_0$ over a certain period of time Δt.

The length of the *AB* trajectory segment traveled by the point since the start of the time count is called ***the path length*** ΔS and is a scalar value.

The path during this time is the distance along a curved trajectory from position *A* to position *B*, if the direction of movement did not change. In the general case $\Delta S \neq |\Delta\vec{r}|$. If point *A* is arbitrarily small distance from *B*, then displacement and path are denoted by symbols of infinitesimally small values and, respectively. In this case, their modules are equal: $|d\vec{r}| = dS$. The time of movement between these two positions will be infinitesimally small dt.

2. Average and instantaneous speeds

Movement speed characterizes the direction and speed of body movement.

The ratio of movement to the time during which this movement occurred determines ***the average speed of movement:***

$$\langle \vec{\upsilon} \rangle = \frac{\Delta\vec{r}}{\Delta t}. \tag{1.2}$$

The direction of the average speed coincides with the direction of movement.

When moving in a straight line $|\Delta\vec{r}| = \Delta S$, where

$$\langle \upsilon \rangle = \frac{\Delta S}{\Delta t}. \tag{1.3}$$

Let's call the speed (instantaneous speed) of the point in position A the limit to which the ratio $\Delta\vec{r}/\Delta t$ at Δt, which goes to zero, goes:

$$\vec{\upsilon} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \ . \qquad (1.4)$$

Therefore, the speed at point *A* is equal to the first time derivative of the radius vector. Vectors $d\vec{r}$ and $\vec{\upsilon}$ are directed tangentially to the trajectory of motion. Since $|d\vec{r}| = dS$, then

$$\upsilon = \frac{dS}{dt} \, . \qquad (1.5)$$

Speed υ is measured in meters per second (m/s).

It follows from formula (1.5) that $dS = \upsilon dt$. Integrating this expression over time within the limits from t to $t + \Delta t$, we get the length of the path traveled by the point in time Δt:

$$S = \int_{t}^{t+\Delta t} \upsilon dt \, . \qquad (1.6)$$

The speed $\vec{\upsilon}$ can be decomposed into three components along the axes of the Cartesian coordinate system:

$$\vec{\upsilon} = \upsilon_x \vec{i} + \upsilon_y \vec{j} + \upsilon_z \vec{k} \ , \qquad (1.7a)$$

moreover,

$$\upsilon_x = \frac{dx}{dt}; \quad \upsilon_y = \frac{dy}{dt}; \quad \upsilon_z = \frac{dz}{dt}, \quad \upsilon = \sqrt{\upsilon_x^{\ 2} + \upsilon_y^{\ 2} + \upsilon_z^{\ 2}} \ . \qquad (1.7.б)$$

A material point can participate simultaneously in movements in different directions. For example, a body thrown at an angle to the horizon participates simultaneously in horizontal and vertical movements. There is a principle of independence of movements, according to which each movement occurs independently of others. Therefore, the speed of the resulting movement at the given moment of time is the vector sum of the speeds of the component movements:

$$\vec{\upsilon} = \sum_{i=1}^{n} \vec{\upsilon}_i \ . \qquad (1.8)$$

3. Acceleration and its components

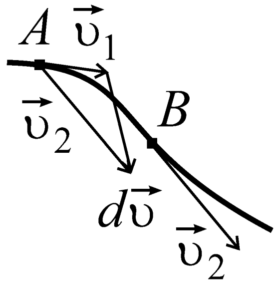


Fig. 1.4

In general, the speed can change over time according to some law $\vec{\upsilon}(t)$. Velocity is a vector. Therefore, it can change in magnitude, in direction, or simultaneously in magnitude and direction. The characteristic of the rapidity of the speed change is the vector value – ***acceleration*** $\vec{a}$.

Consider the movement of a point along a curved trajectory (Fig. 1.4). In the figure $\vec{\upsilon}_1$, denotes the speed of the point in position *A*, $\vec{\upsilon}_2$ is the speed of the point in position *B*, $d\vec{\upsilon}$ is the change in speed during movement from *A* to *B*. Acceleration $\vec{a} = \frac{d\vec{\upsilon}}{dt}$.

Over time Δt, the point moved from one position to another, the velocity vector changed by $\Delta\vec{\upsilon}$. ***The average acceleration*** is determined by equality

$$\langle\vec{a}\rangle = \frac{\Delta\vec{\upsilon}}{\Delta t}. \qquad (1.9)$$

The direction of the average acceleration coincides with the direction of $\Delta\vec{\upsilon}$.

Acceleration is the limit to which the ratio $\Delta\vec{\upsilon}(t)/\Delta t$ at Δt, which goes to zero, goes to:

$$\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{\upsilon}}{\Delta t} = \frac{d\vec{\upsilon}}{dt} = \frac{d^2\vec{r}}{dt^2}. \qquad (1.10)$$

Thus, ***acceleration*** is the first derivative of velocity with time or the second derivative of radius vector with time. Since $|d\vec{r}| = dS$, then

$$a = \frac{d^2S}{dt^2}. \qquad (1.11)$$

The direction of acceleration coincides with the direction of change of speed $d\vec{\upsilon}$.

Acceleration is measured in *meters per second squared* (m/s^2).

Full acceleration can be divided into two components (Fig. 1.5):

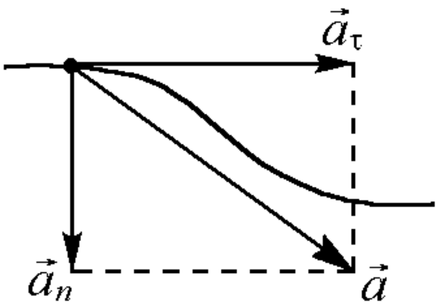


Fig. 1.5

Tangential acceleration a_τ - characterizes the change in the speed module and is directed tangentially to the trajectory:

$$a_\tau = \frac{d\upsilon}{dt}. \qquad (1.12)$$

Normal (centripetal) acceleration a_n - characterizes the change in speed in the direction and directed perpendicularly to the tangent to the center of curvature of the trajectory:

$$a_n = \frac{\upsilon^2}{R}. \qquad (1.13)$$

The total acceleration of a point is the geometric sum of the tangential and normal components (Fig. 1.5):

$$\vec{a} = \vec{a}_\tau + \vec{a}_n, \qquad (1.14)$$

and the point acceleration module

$$a = \sqrt{a_\tau^{\,2} + a_n^{\,2}}. \qquad (1.15)$$

4. Traffic classification

Depending on the tangential and normal components of acceleration, the movement of a point can be:

1. $a_\tau = 0$, $a_n = 0$ is ***uniform rectilinear***: movement in which the velocity vector remains constant ($\vec{\upsilon} = \text{const}$). The path with this movement in time *t* is equal to

$$S = \upsilon \cdot t\ . \tag{1.16}$$

2. $a_\tau = const = a$, $a_n = 0$ – ***uniformly accelerated rectilinear***: motion in which the velocity value increases according to a linear law depending on the time of motion, remaining constant in direction. In the general case of movement with an initial speed $|\vec{\upsilon}_0| = \upsilon_0$, the speed at any moment in time can be calculated, knowing the acceleration a:

$$\upsilon = \upsilon_0 + at\ . \tag{1.17}$$

The path with uniformly accelerated motion in time *t* is equal to

$$S = \int_0^t \upsilon dt = \int_0^t (\upsilon_0 + at) dt = \upsilon_0 t + \frac{at^2}{2}\ . \tag{1.18}$$

3. $a_\tau = f(t)$, $a_n = 0$ – ***rectilinear motion with variable acceleration***.

4. $a_\tau = 0$, $a_n = const$ is ***uniform movement in a circle***. When $a_\tau = 0$ the speed does not change by the module, but changes only by direction. It follows from the formula $a_n = const = \dfrac{\upsilon^2}{R}$ that $R = const$.

5. $a_\tau = const$, $a_n \neq 0$ is ***uniform curvilinear*** motion.

Lecture 3
DYNAMICS OF PROGRESSIVE MOVEMENT. FORCES IN MECHANICS

Plan

1. Newton's 1st law. Inertial reference systems.
2. Mass. Power. Newton's 2nd law.
3. Newton's 3rd law.
4. Impulse of a material point. Newton's 2nd law in impulse form.
5. Law of conservation of momentum.
6. Center of inertia (mass) of the system of bodies.
7. Forces in mechanics.
8. Galileo's mechanical principle of relativity.

1. Newton's 1st law. Inertial reference systems

The basis of classical mechanics are three laws of Newton (1642-1727), formulated in his work "Mathematical Principles of Natural Philosophy", published in 1687.

Newton's 1st law states: There are such frames of reference relative to which a body maintains a state of rest or uniform rectilinear motion until the action of other bodies brings it out of this state.

Newton's 1st law is called the law of inertia. It is not performed in every frame of reference: there is an object in a train carriage. If the train is moving uniformly in a straight line, then the object is at rest. When the train starts moving with acceleration, the thing will move relative to the car without any action from other bodies. Therefore, this system moving with acceleration is not inertial.

Reference systems, relative to which Newton's 1st law is fulfilled, are called ***inertial***. Strictly speaking, inertial systems do not exist in nature, this is an idealization. But there are systems that can be called inertial with great accuracy. (Heliocentric system with the center of reference in the Sun). Every system moving with a constant velocity relative to an inertial one is also inertial. So, it can be argued that when there is one inertial frame of reference, there can be many of them.

2. Mass. Power. Newton's 2nd law

Moving bodies "resist" a change in their speed in different ways, that is, they have different inertia. Mass m is the measure of inertia of bodies. In addition, mass is a measure of the gravitational interaction of bodies (a measure of gravity). It was experimentally established that inert and gravitational masses do not differ from each other. The unit of mass is kilogram (kg). The range of masses in nature is very wide. For example, the mass of an electron is $9{,}1\cdot 10^{-31}$кг, and the mass of our Galaxy is $2{,}2\cdot 10^{41}$кг.

Mass is an additive quantity. The mass of the body is equal to the sum of the masses of individual parts of the body, and the mass of the system is equal to the sum of the masses of the material points (bodies) that make up this system.

Force $\vec{F}$ is a measure of the interaction of bodies (the action referred to in Newton's 1st law). As a result of the action of forces, bodies either gain acceleration or are deformed. Force is a vector quantity. The force vector is determined by the modulus, direction and point of application. The unit of force is the newton (N).

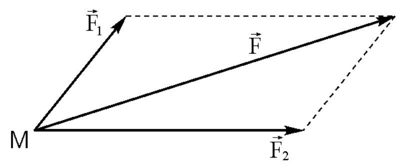


Fig. 1.6

If several forces act on the body, then their action on the body can be replaced by the action of one force $\vec{F}$ equal to their geometric sum:

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i , \qquad (1.19)$$

where $\vec{F}$ is the uniform force.

To combine forces means to find their equivalent $\vec{F}$. This operation is easiest to do in the case of two forces $\vec{F}_1$ and $\vec{F}_2$ applied to one point. The vector $\vec{F}$ is directed along the diagonal of the parallelogram built on the vectors $\vec{F}_1$ and $\vec{F}_2$ (Fig. 1.6). If the body is acted upon by forces applied to different parts of the body, then in order to find the equivalent force, they must be transferred to one point, and then added in pairs.

Newton's second law is the basic law of the dynamics of translational motion, describing the change in motion of a completely rigid body under the action of a force.

Newton's 2nd law: the acceleration that a body acquires is directly proportional to the force applied to it and inversely proportional to the body's mass. The direction of acceleration coincides with the direction of the applied force:

$$\vec{a} = \frac{\vec{F}}{m} . \qquad (1.20)$$

If n forces act on the body, then the force $\vec{F}$ in expression (1.20) means the equivalent of all these forces (see 1.19).

Newton's second law is valid only in inertial frames of reference. The first follows from Newton's second law as a special case $\vec{F} = 0$. Then $\vec{a} = \frac{d\vec{\upsilon}}{dt} = 0 \;\rightarrow\; \vec{\upsilon} = const$. Then But this is nothing but a mathematical record of Newton's First Law. That is $\vec{\upsilon} = const$, when $\vec{F} = 0$.

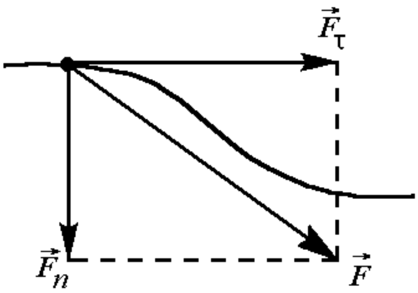


Fig. 1.7

In mechanics, the principle of independence of the action of forces is of great importance: the acceleration given to a body under the simultaneous action of several forces is equal to the sum of accelerations that each force would give to this body, acting separately.

According to this principle, force and acceleration can be decomposed into components. For example (Fig. 1.7), a force acts on a point $\vec{F} = m\vec{a}$. Let's decompose the force into two components: tangential $\vec{F}_\tau = m\vec{a}_\tau$ and normal $\vec{F}_n = m\vec{a}_n$. The force can be found as $\vec{F} = \vec{F}_\tau + \vec{F}_n$, or in scalar form taking into account expressions (1.12) and (1.13)

$$F = \sqrt{F_\tau^2 + F_n^2} = m\sqrt{a_\tau^2 + a_n^2} = m\sqrt{\left(\frac{d\upsilon}{dt}\right)^2 + \left(\frac{\upsilon^2}{R}\right)^2} .$$

3. Newton's 3rd law

This law reflects the fact that the action of one body on another has the character of interaction.

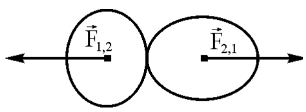


Fig. 1.8

A force $\vec{F}_{2,1}$ acts on body 1 from the side of body 2, and at the same time, a force $\vec{F}_{1,2}$ of equal magnitude but opposite direction acts on body 2 from the side of body 1. Using fig. 1.8, you can write:

$$\vec{F}_{1,2} = -\vec{F}_{2,1} , \qquad (1.21)$$

This equality means that the bodies act on each other with forces directed along the same straight line, equal in absolute value and opposite in direction.

Let's pay attention to the fact that two forces are applied to different bodies, so finding them "equivalent" is meaningless.

4. Impulse of a material point. Newton's 2nd law in impulse form

The momentum of a body (material point) is called a vector $\vec{p}_i$ equal to the product of the mass of the body (point) m_i times its speed $\vec{\upsilon}_i$:

$$\vec{p}_i = m_i \vec{\upsilon}_i \, . \tag{1.22}$$

Impulse is a vector quantity that has the direction of speed.

The unit of impulse is kg m/s. This unit does not have a special name.

The vector sum of the impulses of material points (bodies) of this ***system is the impulse*** $\vec{p}$ of the system:

$$\vec{p} = \sum_{i=1}^{n} \vec{p}_i = \sum_{i=1}^{n} m_i \vec{\upsilon}_i \, . \tag{1.23}$$

Let's write the equality expressing Newton's second law and replace the acceleration according to its definition, taking into account that $m = const$

$$\vec{F} = m\vec{a} = m\frac{d\vec{\upsilon}}{dt} = \frac{d(m\vec{\upsilon})}{dt},$$

where $\vec{p} = m\vec{\upsilon}$ is the momentum of a material point (body).

$$\vec{F} = \frac{d\vec{p}}{dt}, \qquad \vec{F}dt = d\vec{p} \, . \tag{1.24}$$

This is the expression of Newton's second law through momentum. Expression (1.24) is called the equation of motion of a material point.

The magnitude $\vec{F}dt$ is called ***force impulse***. According to Newton's second law in impulse form: the change in momentum of a material point over a period of time dt is equal to the impulse of the force acting on the material point over the same period of time.

5. Law of conservation of momentum

Let's introduce some concepts:

A mechanical system is a set of material points (solid bodies).

Internal forces are forces with which the bodies of a given system interact with each other.

External forces are forces with which bodies outside the system act on the bodies of this system.

A closed (isolated) system is a system that is not affected by external forces.

Consider a system consisting of n bodies (points). Each body of the system can be affected by internal (with respect to this system) and external forces acting on the part of bodies that are not part of this system. Let's write down Newton's second law for each body of the system:

$$\vec{f}_i + \vec{F}_i = \frac{d\vec{p}_i}{dt}, \quad (1.25)$$

where $\vec{f}_i$ is the net effect of all internal forces acting on the ith body of the system;

$\vec{F}_i$ - uniform action of all external forces acting on this body;

$\vec{p}_i$ is the momentum of this body.

It is necessary to write n such equations. For a system of bodies, adding these equations term by term, we obtain

$$\sum_{i=1}^{n} \vec{f}_i + \sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} \frac{d\vec{p}_i}{dt} = \frac{d\vec{p}}{dt}, \quad (1.26)$$

According to Newton's 3rd law, the geometric sum of internal forces is zero $\sum_{i=1}^{n} \vec{f}_i = 0$.

Equation (1.26) will be rewritten in the form $\sum_{i=1}^{n} \vec{F}_i = \sum_{i=1}^{n} \frac{d\vec{p}_i}{dt} = \frac{d\vec{p}}{dt}$

If the system is closed, then $\vec{F} = \sum_{i=1}^{n} \vec{F}_i = 0$. So, for such a system $\frac{d\vec{p}}{dt} = 0$ and

$$\vec{p} = \sum_{i=1}^{n} m_i \vec{\upsilon}_i = const. \quad (1.27)$$

We obtained ***the law of conservation of momentum:*** the momentum of a closed system of bodies is a constant value, that is, it does not change over time.

Momentum is conserved even for an open system if the net effect of all external forces is zero.

In projections on the axis of the Cartesian coordinate system, the law of conservation of momentum can be written as follows:

$$p_x = const \quad \text{at} \quad F_x = 0,$$
$$p_y = const \quad \text{at} \quad F_y = 0, \quad (1.28)$$
$$p_z = const \quad \text{at} \quad F_z = 0.$$

If the system of bodies is not closed, but the projection of external forces on some axis is zero, then the projection of momentum on this axis is preserved.

The law of conservation of momentum, related to the symmetry of space (homogeneity of space), is universal in nature, that is, it is a fundamental law of nature.

Lecture 4
DYNAMICS OF PROGRESSIVE MOVEMENT. FORCES IN MECHANICS (CONTINUED)

6. Center of inertia (mass) of the system of bodies.

The center of inertia (center of mass) of a system of material points is an imaginary geometric point that characterizes the distribution of masses in this system. The radius vector $\vec{r}_c$ of the center of mass of the system n of material points is determined by equality

$$\vec{r}_c = \frac{\sum\limits_n m_i \vec{r}_i}{\sum\limits_n m_i} = \frac{\sum\limits_n m_i \vec{r}_i}{m}, \qquad (1.29)$$

where m_i – and $\vec{r}_i$ – respectively, the mass and radius vector of the *i*-th point;

$m = \sum\limits_n m_i$ is the mass of the system.

The center of mass can also be outside the body (for example, the center of mass of a hoop is in its geometric center).

The speed of the center of mass

$$\vec{\upsilon}_c = \frac{d\vec{r}_c}{dt} = \frac{\sum\limits_n m_i \frac{d\vec{r}_i}{dt}}{m} = \frac{\sum\limits_n m_i \vec{\upsilon}_i}{m}. \qquad (1.30)$$

Let's rewrite equation (1.30) in the form:

$$m\vec{\upsilon}_c = \sum_n m_i \vec{\upsilon}_i$$

Taking into account that $\vec{p}_i = m_i \vec{\upsilon}_i$, and $\sum\limits_n \vec{p}_i$ is the momentum $\vec{p}$ of the system:

$$\vec{p} = m\vec{\upsilon}_c, \qquad (1.31)$$

that is, the momentum of the system is equal to the product of the mass of the system by the speed of its center of mass.

Substitute (1.31) into equation 2 of Newton's law in impulse form $\vec{F} = \frac{d\vec{p}}{dt}$ and obtain the ***law of motion of the center of mass***:

$$m\frac{d\vec{\upsilon}_c}{dt} = \vec{F}. \qquad (1.32)$$

The center of mass of the system moves as if the entire mass of the system is concentrated in it, and the net force of all forces acting on the system is applied to it.

This law makes it possible to move from the dynamics of a material point to the dynamics of a solid body. Indeed, a solid body can be considered as a system of material points. At the same time, the point of application of the forces acting on the body is the center of mass, and the laws of motion have the same form as for a material point.

It can be seen from (1.32) that in a closed system the velocity of the center of mass became The center of mass of a closed system is either at rest or moving uniformly in a straight line.

7. Forces in mechanics

1. **Gravity.**

All bodies (particles) in nature are subject to gravitational interaction. It manifests itself in the attraction (gravitation) of bodies (particles) to each other with forces called gravitational. Gravitational forces are subject to Newton's ***law of universal gravitation***, according to which all bodies are attracted to each other with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

$$F = G\frac{m_1 m_2}{r^2} \; . \tag{1.33}$$

The proportionality coefficient G is called the ***gravitational constant*** and is equal to the gravitational force that acts between two material points located at a distance of 1 m from each other, with masses of 1 kg each. The value of G, obtained by modern methods, is taken as $6{,}6745 \cdot 10^{-11}$ N·m^2/kg^2. The small value of G shows that the gravitational interaction is significant only in the case of large masses.

A force acts on any body with a mass m near the Earth, due to which it (deprived of support or suspension) will begin to move with the acceleration of free fall $\vec{g}$. This force is called ***the force of gravity***, and it is equal to

$$\vec{P} = m\vec{g} \; . \tag{1.34}$$

The acceleration of free fall can be determined from the law of universal gravitation. According to the law of universal gravitation, a body with a mass m lying near the surface of the Earth is attracted by the Earth with a force

$$F = G\frac{mM_з}{R^2} , \tag{1.35}$$

where M and R are the mass and radius of the Earth, respectively.

Comparing (1.34) and (1.35), we find

$$g = G\frac{M_з}{R^2} . \tag{1.36}$$

The acceleration of free fall at the level of the Earth's surface at a given geographic latitude is the same for all bodies: at the pole 9.83 m/s^2, at the equator $g = 9.78$ m/s^2, at latitude 45° = 9.81 m/s^2.

The acceleration of free fall depends on the height above the Earth's surface, decreases by approximately 0.03% for every 1 km of ascent. At an altitude of 5000 km $g = 3.08$ m/s^2, and at altitude 50000 km $g =$ 0.13 m/s^2.

Space speeds. The speed at which a body moves in a circular orbit around the Earth under the influence of universal gravity is called the first cosmic speed υ_1. The body, which was given the first cosmic speed, will become an artificial satellite of the Earth. At the same time, the satellite will move with a constant velocity and centripetal acceleration $a_ц = g$. Neglecting the height of the satellite above the Earth's surface and using the expression $a_ц = \frac{\upsilon^2}{R}$, into which R we substitute the Earth's radius instead, we obtain

$$\upsilon_1 = \sqrt{gR} = \sqrt{9{,}8 \cdot 6{,}4 \cdot 10^6} \approx 8 \cdot 10^3 \text{ м/с}.$$

The speed that must be given to the body so that it overcomes the attraction of the Earth and begins to move in a circular orbit around the Sun is called the second cosmic speed υ_2 = 11.2 km/h

The speed that must be given to the body so that it overcomes the attraction of the Sun and leaves the Solar System is called the third cosmic speed υ_3 = 16.7 km/h.

2. Forces of elasticity.

Under the action of external forces or fields, the body can change its shape, that is, deform. If the deformation disappears after the cessation of external actions, then such deformation is called ***elastic***. The deformation that remains after the load is removed is called ***plastic***. Deformations are reduced to stretching (compression) and shear. During deformations, the relative arrangement of atoms or molecules changes.

Stretching (compression) deformation is characterized by absolute elongation $\Delta\ell$:

$$\Delta\ell = \ell - \ell_0 \ , \qquad (1.37)$$

where ℓ_0 ℓ and are the length of the sample before and after deformation, respectively. During stretching $\Delta\ell > 0$, during compression $\Delta\ell < 0$.

Relative elongation is called magnitude

$$\varepsilon = \frac{\Delta\ell}{\ell_0} \ . \qquad (1.38)$$

If, under the action of the applied force, the atoms are displaced from their equilibrium positions in the crystal at a distance smaller than the interatomic distances, then elastic forces arise that return the atoms to the equilibrium position.

Mechanical stress σ is the ratio of the force *F* that stretches (compresses) the sample to the size of the cross section of the sample *S*, perpendicular to the elastic force, i.e.

$$\sigma = \frac{F}{S} \qquad (1.39)$$

The unit of mechanical stress is the pascal (Pa).

At small elastic deformations, Hooke's law applies: mechanical stress is directly proportional to relative elongation:

$$\sigma = E \cdot \varepsilon \ , \qquad (1.40)$$

The proportionality factor E is called the ***modulus of elasticity, or Young's modulus***. It can be seen from (1.40) that the Young's modulus is determined by the stress that creates a relative elongation equal to unity. Young's modulus depends on the material of the sample.

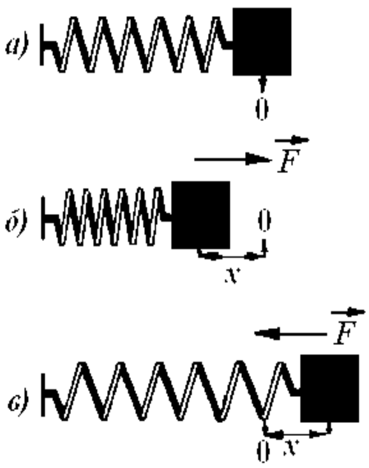


Fig. 1.9

In the area of elastic deformation of the body, there is a linear relationship between the deformation and the magnitude of the elastic force F (Fig. 1.9):

$$F = -kx \ .$$

The value k is usually called ***the stiffness of the body***, or ***the stiffness coefficient***. The minus sign means that the elastic force is directed in the direction of decreasing deformation.

The greatest stress at which no noticeable residual deformation occurs is called the elastic limit. At loads exceeding the elastic limit, Hooke's law is not fulfilled. Bodies that have a small elastic limit (bodies made of lead, soft clay, wax) are called plastic, others are elastic (steel, glass).

3. Friction forces.

Frictional forces arise on the surface of contacting bodies and prevent their relative movement. There are three types of friction: static friction, sliding friction, and rolling friction.

If the relative speed of the bodies in contact is zero, then friction at rest is observed. ***Friction forces*** in this case can take any value from zero to some maximum value depending on the module and direction of the applied external force.

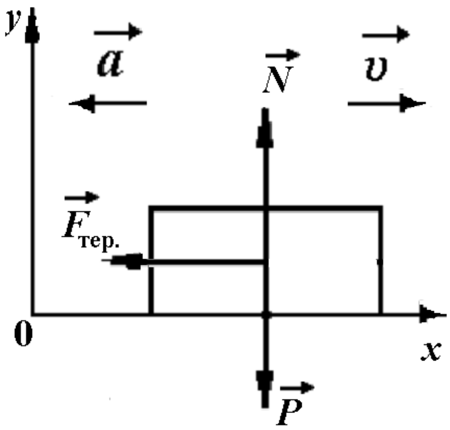


Fig. 1.10

The force of sliding friction arises during the relative movement of contacting bodies and is always directed along the contact boundary of the bodies opposite to the relative speed.

The French physicists G. Amonton (1663-1705) and S. Coulomb (1736-1806) experimentally established the following law: *the force of sliding friction is proportional to the force of normal pressure, or the force of the support reaction* N :

$$F_{\text{тер}} = \mu \cdot N \ .$$

The value μ is called the coefficient of friction. For this pair of surfaces μ is a constant value, depending on the type and quality of the contact surfaces. The coefficient of sliding friction also depends on the relative speed of the bodies. At low speeds, it can be assumed that the coefficient of sliding friction is equal to the coefficient of rest friction.

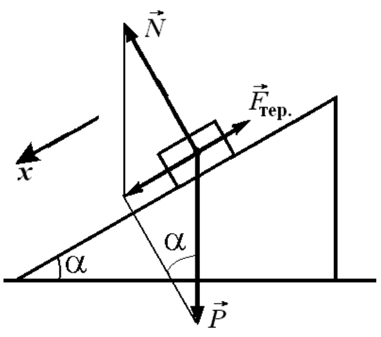


Fig. 1.11

Consider a body moving along a horizontal plane under the action of friction only (Fig. 1.10). The force of sliding friction is always directed against the relative motion. The acceleration that this force provides to the body is also directed against the movement, that is, negative. From Newton's second law $a = -\dfrac{F_{\text{тр}}}{m}$.

For this case $N = mg$ and $F_{\text{тер}} = \mu \cdot mg$.

Consider a body moving along an inclined plane under the action of friction only (Fig. 1.11). Let's get an expression for the force of friction. According to (1.42) $F_{тер} = \mu \cdot N$. It can be seen from the figure that $N = P \cdot \cos\alpha = mg \cdot \cos\alpha$, taking this into account $F_{тер} = \mu \cdot mg \cos\alpha$.

The force of friction can be reduced if sliding friction is replaced by rolling friction, which, for example, is realized in ball bearings. ***The force of rolling friction*** is inversely proportional to the radius of the rolling body:

$$F_{тр} = \frac{f_k \cdot N}{r}, \quad (1.43)$$

where f_k is the coefficient of rolling friction.

8. Galileo's mechanical principle of relativity

Consider two inertial systems. One of them *K′* moves relative to the other *K* (Fig. 1.12) with a constant speed $\vec{\upsilon}$. The axes of the Cartesian reference system will be denoted x', y', z' by *x, y, z*, respectively. For simplicity, we will assume that the movement occurs along the axis x, while assuming that at the initial moment of time *t=0*, both systems coincided. Let's take some material point *A*. Let's consider its coordinates relative to both of these systems at some point in time *t* and find the connection between these coordinates.

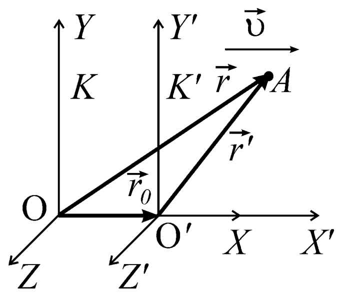


Fig.1.12

In classical mechanics, time is considered absolute, that is, the passage of time in different reference systems is the same. In our case, this means that *t= t'*.

Let us denote the radii-vectors of the point *A* in the system *K* by $\vec{r}$, and in the system *K′* by $\vec{r}\,'$, the radius-vector of the point O' – $\vec{r}_0$, From the given construction we find $\vec{r} = \vec{r}\,' + \vec{r}_0$:, We take into account that $\vec{r}_0 = \vec{\upsilon}t'$. So, the desired transformation is:

$$\vec{r} = \vec{r}\,' + \vec{\upsilon}t', \qquad t = t'. \quad (1.44)$$

Or in Cartesian coordinates:

$$x = x' + \upsilon t', \quad y = y', \quad z = z', \quad t = t'. \quad (1.45)$$

This is *the Galilean transformation*, according to which, knowing the coordinates of a point in the moving system *K′*, we find the coordinates of the same point relative to the system *K*. The reverse transformation is also quite obvious:

$$\vec{r}\,' = \vec{r}_0 - \vec{\upsilon}t, \quad t' = t. \quad (1.46)$$

Or

$$x' = x - \upsilon t, \quad y' = y, \quad z' = z, \quad t' = t. \quad (1.47)$$

Let's establish the connection between the velocities of the material point in the *K* and *K'* systems. To do this, we find the derivative of equality $\vec{r} = \vec{r}\,' + \vec{\upsilon}t'$ in time, taking into account that *t=t'*:

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \vec{\upsilon} \Rightarrow \vec{u} = \vec{u}' + \vec{\upsilon} . \qquad (1.48)$$

Here $\vec{u}$ is the velocity of the material point in the *K* system;

$\vec{u}'$ – velocity of the same point in the *K'* system;

$\vec{\upsilon}$ is the speed of system *K'* relative to system *K*.

To find the relationship between the accelerations in these two inertial systems, we find the time derivative from the equality $\vec{u} = \vec{u}' + \vec{\upsilon}$. At the same time, we take into account that $\vec{\upsilon} = const$, $\frac{d\vec{\upsilon}}{dt} = 0$. Then:

$$\frac{d\vec{u}}{dt} = \frac{d\vec{u}'}{dt} \Rightarrow \vec{a} = \vec{a}' . \qquad (1.49)$$

Therefore, the acceleration of a material point relative to the inertial system *K* (stationary) is the same as the acceleration relative to the system *K'*.

The equality of the accelerations of the same body in different inertial frames of reference also implies the equality of the forces acting on them. The above leads to a conclusion known as the "Mechanical Principle of Relativity" or "***Galilean's Principle of Relativity***": No mechanical experiments conducted on an inertial system can establish whether this system is moving uniformly in a straight line or is at rest relative to another inertial system.

Example: If a train car were to move uniformly in a straight line relative to a railway station along an ideal track with closed windows, soundproof walls, etc., we would not be able to determine by any mechanical experiments whether we are really moving or standing still.

Let us give a stricter formulation of Galileo's principle of relativity: *the laws of mechanics are the same in all inertial systems*. Or: *The laws of mechanics are invariant with respect to Galilean transformations*.

The mechanical principle of relativity reflects quite certain properties of space and time, in particular the absoluteness of the passage of time.

Lecture 5
Topic 2
ENERGY, WORK, POWER

Plan

1. Energy, work, power.
2. Conservative and dissipative forces.
3. Kinetic energy.
4. Potential energy.
5. Law of conservation of energy.
6. Graphic interpretation of energy.

1. Energy, work, power

Energy is one of the most important, fundamental concepts of physics. ***Energy*** is a universal measure of various forms of movement and interaction. Different forms of energy are associated with different forms of movement: mechanical, thermal, electromagnetic, etc. Mechanical energy is the simplest form of energy. Mechanical energy characterizes the system from the point of view of possible quantitative and qualitative transformations in it, the ability of the system to perform work.

A change in the mechanical motion of a body is caused by the action of other bodies on it. In order to quantitatively characterize the process of energy exchange between interacting bodies, the concept of ***work of force*** is introduced in mechanics.

Elementary work dA at an infinitesimal movement $d\vec{r}$ of a body under the action of a force $\vec{F}$ is understood as a scalar product $\vec{F}$ and $d\vec{r}$:

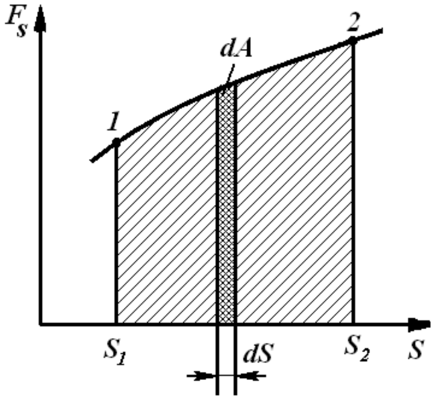


Fig. 2.1

$$dA = \vec{F}d\vec{r} = Fdr\cos\alpha . \qquad (2.1)$$

Here α is the angle between the direction of the force and the displacement. The displacement is so small that the force when the body moves along the corresponding trajectory remains unchanged both in magnitude and direction. At the same time, the path and displacement by module are equal $dS = |d\vec{r}|$, so the work can be written in the form:

$$dA = FdS\cos\alpha . \qquad (2.2)$$

When it is necessary to find work on a segment of the path 1-2, along which the force changes, then the entire path is divided into such small segments that the force can be considered constant on each of them (Fig. 2.1). The work of the force on the final segment of the path from point 1 to point 2 is equal to the algebraic sum of elementary work on separate infinitesimally small segments. Such an amount is expressed as an integral:

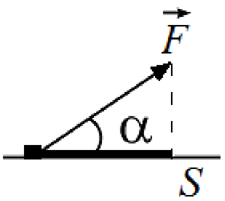


Fig. 2.2

$$A = \int_1^2 \vec{F}d\vec{r} = \int_1^2 FdS\cos\alpha . \qquad (2.3)$$

To calculate this integral, it is necessary to know the dependence of the force F on the path S. If this dependence is represented graphically, then the work A is determined on the graph by the area of the shaded figure (Fig. 2.1). If the body moves in a straight line under the action of a constant force $\vec{F}$ directed at an angle α to the movement

(Fig. 2.2), then the mechanical work is equal to the product of the modulus of the force by the modulus of displacement of the point (body) S and the cosine of the angle between the direction of the force and the displacement:

$$A = FS\cos\alpha \,. \qquad (2.4)$$

Work is measured in joules (J).

Work is an algebraic quantity. The work is positive if $\alpha < \pi/2$, negative if $\alpha > \pi/2$, and equal to zero if $\alpha = \pi/2$.

Not only the amount of work that a certain machine can perform, but also the time during which this work can be performed is important for characterizing the performance of various machines. The intensity of work performance is characterized by ***power N***, which is defined as the ratio of the work performed to the execution time:

$$N = \frac{dA}{dt} \,. \qquad (2.5)$$

During time dt, force $\vec{F}$ performs work $\vec{F}d\vec{r}$, and ***power at a given moment of time***

$$N = \frac{\vec{F}d\vec{r}}{dt} = \vec{F}\vec{\upsilon} = F\upsilon\cos\alpha \,, \qquad (2.6)$$

that is, it is equal to the scalar product of the force vector by the velocity vector at which the point to which the force is applied moves.

Power is measured in watts (W). In practice, the non-system unit of power - "horsepower" is quite often used. 1 hp = 736 W.

Any mechanism that performs work must obtain energy at the expense of which this work is performed. Part of this energy is spent on overcoming the frictional forces that always operate in any mechanism. The ratio of the power that the mechanism transmits to the consumer to the total power supplied to the mechanism is called the ***coefficient of useful action (k.k.d.)*** of this mechanism. If the power supplied to the mechanism is denoted by , and the power delivered by the mechanism to the consumer is denoted by N_2, then k.k.d. η mechanism

$$\eta = \frac{N_2}{N_1} \cdot 100\% \,. \qquad (2.7)$$

Since power losses are inevitable in any mechanism, the k.k.d. is always less than one; it is usually given as a percentage.

2. Conservative and dissipative forces

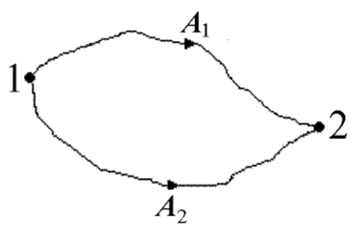


Fig. 2.3

Conservative forces are called forces whose work does not depend on the shape of the path (on the trajectory) along which the work is performed, but is determined only by the initial and final positions of the body. When the force does not meet the specified condition, it is called ***dissipative*** (or dissipative).

Yes, according to the definition of conservatism of forces $A_1 = A_2$ (Fig. 2.3). A change in the direction of movement causes a change in the sign of the work of the conservative force $A_{1-2} = -A_{2-1}$. The work of conservative forces along a closed circuit is zero: $\oint_L \vec{F}d\vec{r} = A_{1-2} + A_{2-1} = -A_{2-1} + A_{2-1} = 0$.

Examples of such forces in mechanics are the forces of gravity and elasticity. An example of dissipative forces is the force of friction. A system in which only conservative forces (external and internal) act is called ***conservative***.

3. Kinetic energy

The subject of physics is the study of various forms of motion of matter. The measure of the movement of matter is energy. The energy of the system changes in the process of performing work. That is, work can be defined as a process in which the energy of the system changes under the action of forces, and as a quantitative measure of this change. In mechanics, two types of energy are distinguished - ***kinetic and potential***.

Energy, like work, is measured in joules (J).

Kinetic energy W_k is the energy of a moving body.

The force $\vec{F}$ acting on a stationary body and causing its motion performs elementary work $dA = \vec{F}d\vec{r}$. The tangential component F_τ of the force F changes the numerical value of the body's velocity. According to Newton's second law $F_\tau = m\frac{d|\vec{\upsilon}|}{dt}$, therefore $dA = m\frac{d\upsilon}{dt}dr$. Since $\upsilon = \frac{dr}{dt}$, then $dA = m\upsilon d\upsilon$. The energy of a moving body increases by the amount of work expended, i.e. $dA = dW_k = m\upsilon d\upsilon$, from where $W_k = \int_0^\upsilon m\upsilon d\upsilon = \frac{m\upsilon^2}{2}$.

The kinetic energy of a material point with a mass moving at a speed of $\vec{\upsilon}_i$

$$W_{ki} = \frac{m_i\upsilon_i^2}{2}. \tag{2.8}$$

It can be seen from formula (2.8) that the kinetic energy depends on the mass of the body and the speed of its movement, therefore, the kinetic energy of the system is a function of the system's state of motion. Kinetic energy is always positive.

When deriving formula (2.8), it was assumed that the motion was considered in an inertial system (otherwise it is impossible to use Newton's laws). In different inertial systems moving relative to each other, the speed of the body and, accordingly, its kinetic energy will be different. Thus, the kinetic energy depends on the choice of frame of reference.

The kinetic energy of a system consisting of material points is equal to the sum of their kinetic energies:

$$W_k = \sum_{i=1}^{n} W_{ki} = \sum_{i=1}^{n} \frac{m_i\upsilon_i^2}{2}. \tag{2.9}$$

The change in kinetic energy of a system of bodies occurs under the influence of various forces acting on all bodies of this system, i.e

$$dW_k = dA, \tag{2.10}$$

where dA is the total work of these forces.

Lecture 6
ENERGY. WORK. POWER (CONTINUED)

4. Potential energy

Fields of conservative (potential) forces are called ***potential***. A body in a potential field has potential energy. When they talk about the potential energy of a body, they always mean the energy of interaction of this body with other bodies, although they don't always talk about it explicitly.

Potential energy W_n is mechanical energy due to the mutual location of bodies in the system (system configuration) and the nature of the forces of interaction between them. The change in system configuration is related only to the state of the system at the beginning and at the end of the process, it does not depend on the intermediate configurations through which the system passed. That is, the change in the potential energy of the system is related to the work of only conservative forces of this system. When positive work is performed by conservative forces, the potential energy of the system decreases. For example, when a stone falls in the Earth's gravitational field, the work of conservative forces is positive, the potential energy decreases.

The change in the potential energy of the system is equal to the work of its conservative forces (internal or external to the system), taken with the opposite sign:

$$dW_n = -dA\ . \tag{2.11}$$

The work of conservative forces is equal to the reduction of potential energy W_n.

Let's rewrite formula (2.11) taking into account $dA = \vec{F}d\vec{r}$:

$$\vec{F}d\vec{r} = -dW_n\,, \tag{2.12}$$

where

$$W_n = -\int \vec{F}d\vec{r} + const\ . \tag{2.13}$$

The potential energy is determined with an accuracy of some constant. To $const = 0$ choose the "zero" reference level - the energy of the body in this position is considered equal to zero. And the energy in other positions is calculated relative to the "zero" level.

For conservative forces from equation (2.12)

$$F = -\frac{dW_n}{dr} \quad \text{or} \quad F_x = -\frac{\partial W_n}{\partial x} \quad F_y = -\frac{\partial W_n}{\partial y} \quad F_z = -\frac{\partial W_n}{\partial z}\,,$$

in vector form

$$\vec{F} = -\left(\frac{\partial W_n}{\partial x}\vec{i} + \frac{\partial W_n}{\partial y}\vec{j} + \frac{\partial W_n}{\partial z}\vec{k}\right) = -\,grad\,W_n\ . \tag{2.14}$$

where $\vec{i}, \vec{j}, \vec{k}$ are coordinates, unit vectors of the coordinate axes.

The force acting on a body in a potential field is equal to the gradient of the body's potential energy taken with the reverse sign.

The specific form of the function W_n depends on the nature of the force field. Example:

1. Potential energy of a body with a mass m raised to a height above the Earth's surface.

Gravity $p = mg$ acts on the body. The potential energy of a body is equal to the work of gravity when a body falls from a height to the Earth's surface $A = ph$:

$$W_n = mgh, \tag{2.15}$$

where h is the height counted from the zero level, for which $W_{n_0} = 0$.

2. Potential energy of a body with a mass m located at the bottom of a mine with a depth of h'.

We take the Earth's surface as the zero level, therefore, the potential energy of the body located at the bottom of the mine

$$W_n = -mgh' \tag{2.16}$$

Since the beginning of the reference (zero level) is chosen arbitrarily, the potential energy can take negative values.

3. Potential energy of an elastically deformed body.

Deformation occurs under the action of a force F which, according to Newton's 3rd law, is equal to the modulus of the elastic force and is directed opposite to it $F = -F_{np} = kx$. Elementary work $dA = Fdx = kx\,dx$, and full work $A = \int_0^x kx\,dx = \frac{kx^2}{2}$ goes to increase the potential energy of the body. Thus, the potential energy of an elastically deformed body

$$W_n = \frac{kx^2}{2}. \tag{2.17}$$

The potential energy of the system is a function of the positional state of the system. It depends only on the configuration of the system and its position relative to external bodies.

5. Law of conservation of total mechanical energy

In the previous lecture, we defined energy as a universal measure of various forms of movement and interaction. Therefore, ***the total mechanical energy of the system*** is equal to the sum of the kinetic and potential energies:

$$W = W_k + W_n. \tag{2.18}$$

Consider a system consisting of bodies (points). Internal and external conservative forces and external non-conservative forces can act on each body of the system. Let's write down Newton's second law for each body of the system:

$$m\frac{d\vec{\upsilon}_i}{dt} = \vec{F}_i' + \vec{F}_i + \vec{f}_i, \tag{2.19}$$

where $\vec{F}_i'$ – is the net effect of all internal conservative forces acting on the i-th body of the system;

$\vec{F}_i$ - uniform effect of all external conservative forces acting on this body;

$\vec{f}_i$ is the equivalent of all external non-conservative forces acting on this body.

Moving under the action of forces, bodies (points) move during a time dt interval. Let's multiply each equation scalar by the corresponding displacement:

$$m\frac{d\vec{\upsilon}_i}{dt}d\vec{r}_i = \vec{F}_i'd\vec{r}_i + \vec{F}_i d\vec{r}_i + \vec{f}_i d\vec{r}_i \, ,$$

Given that $d\vec{r}_i = \vec{\upsilon}_i dt$ we will get:

$$m_i(\vec{\upsilon}_i d\vec{\upsilon}_i) = (\vec{F}_i' + \vec{F}_i)d\vec{r}_i + \vec{f}_i d\vec{r}_i$$

It is necessary to write n such equations. For a system of bodies, adding these equations term by term, we obtain:

$$\sum_{i=1}^{n} m_i(\vec{\upsilon}_i d\vec{\upsilon}_i) - \sum_{i=1}^{n}(\vec{F}_i' + \vec{F}_i)d\vec{r}_i = \sum_{i=1}^{n} \vec{f}_i d\vec{r}_i \, . \qquad (2.20)$$

The first term on the left side of equality (2.20), $\sum_{i=1}^{n} m_i(\vec{\upsilon}_i d\vec{\upsilon}_i) = \sum_{i=1}^{n} d(m_i\vec{\upsilon}_i^2/2) = dW_k$ where dW_k – increase in kinetic energy. The second term $\sum_{i=1}^{n}(\vec{F}_i' + \vec{F}_i)d\vec{r}_i$ is equal to the elementary work of internal and external conservative forces, taken with a minus sign, that is, it is equal to the elementary increase in the potential energy of the system dW_n. The right-hand side of equation (2.20) specifies the work dA of external non-conservative forces acting on the system. Thus, we have:

$$d(W_k + W_n) = dA \, . \qquad (2.21)$$

When the system transitions from state 1 to state 2

$$\int_1^2 d(W_k + W_n) = A_{1-2} \, ,$$

The change in the total mechanical energy of the system during the transition from one state to another is equal to the work done by external non-conservative forces. In the absence of nonconservative forces $dA = 0$ and, therefore, from (2.21) it follows that $dW = 0$, and

$$W = W_k + W_n = const \, . \qquad (2.22)$$

This is the law of conservation of energy in mechanics: the total mechanical energy of a conservative system is a constant value. (See definition of conservative system in lecture 5).

The law of conservation of energy follows from the uniformity of time, that is, the independence of the laws of physics from the choice of the beginning of the time countdown.

The application of conservation laws to the solution of mechanical problems allows not to consider the intermediate states of the system, but to immediately compare the initial and final state. This makes it easier and faster to solve problems.

For example, consider idealized impacts - short-term interactions of bodies.

A perfectly elastic central impact of two bodies is an impact in which the bodies bounce off each other, preserving the total kinetic energy.

The masses m_1 and m_2 of these bodies are known, $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ their velocities are directed along the line of their centers. After the impact, the velocities of these bodies $\vec{u}_1$ and $\vec{u}_2$, respectively, are directed along the same line. To solve this problem (that is, to find the velocities $\vec{u}_1$ and $\vec{u}_2$), you can use the laws of conservation of momentum and energy:

$$m_1\vec{\upsilon}_1 + m_2\vec{\upsilon}_2 = m_1\vec{u}_1 + m_2\vec{u}_2,$$

$$\frac{m_1\upsilon_1^{\,2}}{2} + \frac{m_2\upsilon_2^{\,2}}{2} = \frac{m_1u_1^{\,2}}{2} + \frac{m_2u_2^{\,2}}{2}.$$

This system of equations with two unknowns is quite easy to solve. Let's find the velocities of the bodies u_1 and u_2 after the impact:

$$u_1 = \frac{\upsilon_1(m_1 - m_2) + 2m_2\upsilon_2}{m_1 + m_2}, \qquad u_2 = \frac{\upsilon_2(m_2 - m_1) + 2m_1\upsilon_1}{m_1 + m_2}.$$

Absolutely inelastic central impact. After the impact, the bodies stick together and continue to move together at the same speed $\vec{u}$. The total speed $\vec{u}$ can be found by the law of conservation of momentum:

$$m_1\vec{\upsilon}_1 + m_2\vec{\upsilon}_2 = (m_1 + m_2)\vec{u}.$$

With such an impact, part of the mechanical energy is converted into internal energy (that is, into heat). According to the law of energy conservation and transformation, these heat losses can be calculated:

$$\Delta Q = \Delta W = \frac{m_1\upsilon_1^{\,2}}{2} + \frac{m_2\upsilon_2^{\,2}}{2} - \frac{(m_1 + m_2)u^2}{2}.$$

6. Graphic interpretation of energy

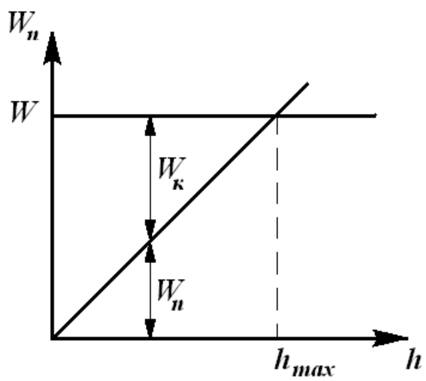


Fig. 2.4

A graph of the dependence of potential energy on some argument is called a ***potential curve***.

Consider only conservative systems:

1. According to (2.15), ***the potential energy of a body with a mass*** m ***raised to a height*** h ***above the Earth's*** surface is equal to $W_n = mgh$. The graph of this dependence is a straight line passing through the origin (Fig. 2.4). The total energy of the body is W (its graph is straight, parallel to the axis h). At height h, the body has potential energy W_n.

Kinetic energy is given by the ordinate between the graph of the potential line and the horizontal line, which specifies the total energy. It follows from the figure: if $h = h_{\max}$, then $W_k = 0$ also $W = W_n = mgh_{\max}$.

2. ***The dependence of the potential energy of elastic deformation*** $W_n = \frac{kx^2}{2}$ ***on the deformation*** x has the form of a parabola (Fig. 2.5), where the graph of the total energy of the body W is a straight line parallel to the abscissa axis. It follows from Fig. 2.5 that as the deformation increases, the potential energy of the body also increases, and the kinetic energy decreases. The abscissa $x_{\max}$ defines the maximum possible stretching deformation of the body, and $x_{\max}$ the maximum possible compression deformation. If $x = \pm x_{\max}$, then $W_k = 0$ and $W = W_n = \frac{kx^2}{2}$. Since the kinetic energy of a body cannot be negative, the potential energy cannot be greater than the total energy. In this case, the body is said to be in a potential well with coordinates $-x_{\max} \le x \le x_{\max}$.

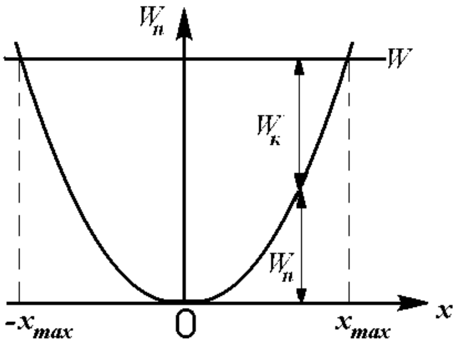


Fig. 2.5

Lecture 7
Topic 3
KINEMATICS AND DYNAMICS OF ROTATIONAL MOVEMENT

Plan

1. Element of angular movement.
2. Angular velocity. Unit of angular velocity. Relationship between linear and angular velocities. Period, frequency, cyclic frequency, relationship between them.
3. Angular acceleration. Unit of angular acceleration.
4. Relationship of normal and tangential acceleration with angular velocity and angular acceleration.
5. Moment of inertia of a material point. The moment of inertia of a rigid body. Moments of inertia of some bodies of regular geometric shape. Steiner's theorem.
6. The moment of strength. Moment of impulse.
7. Kinetic energy of a rotating body.
8. The basic equation of the dynamics of rotary motion. Work with body rotation.
9. Law of conservation of angular momentum.
10. Comparison of translational and rotational motion.

1. Element of angular movement

The rotational motion of a rigid body is a motion in which all points of the body describe circular trajectories, the centers of which are located on the same straight line - the axis of rotation. It is meaningless to talk about the rotational movement of a point relative to the axis on which it lies.

Consider the rotation of a completely rigid body around an axis that does not rotate in space. Separate points of the body will describe circles of different radii, that is, they will travel S different paths in the same time, and have different speeds υ and accelerations a. Therefore, for the rotational movement of the body, it is incorrect to use S, υ, a. But all points of the body will turn to the same angle φ at the same time.

The angle φ of rotation is measured in radians (rads).

The measure of the movement of the body in a short period of time dt is ***the elementary angular displacement*** vector $d\vec{\varphi}$. By modulus, the elementary angular displacement is equal to the angle $d\varphi$ of rotation, and the direction is determined according to the right-hand screw or drill rule: the direction $d\vec{\varphi}$ coincides with the direction of translational movement of the drill tip along the axis, if the drill handle is rotated in the direction of the point movement in a circle (Fig. 3.1).

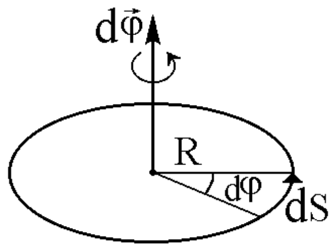


Fig. 3.1

Strictly speaking, $d\vec{\varphi}$ is a pseudo-vector. Vectors related to the direction of rotational motion are also called axial vectors. $d\vec{\varphi}$ in rotational motion it plays the same role as $d\vec{r}$ in translational motion. For the path S during translational motion, the analog is the angle of rotation φ.

2. Angular velocity. Unit of angular velocity. Connection of linear and angular velocities. Period, frequency, cyclic frequency, relationship between them

The analogy between translational and rotational movements continues even further, when defining angular velocity. ***Angular speed*** is the value that characterizes the rate of change of the angle of rotation. When we talk about the movement of a material point, we mean the angle of rotation of the radius vector drawn from the axis of rotation to this point (Fig. 3.1). In the general case, the trajectory of movement can be any curve, but it should be borne in mind that a sufficiently small segment of such a trajectory can be considered an arc of some kind of circle along which the point moves at the given moment of time. Along with the movement of the point, there is a change in the position and radius of the vector drawn from the center of this circle to the moving point. Of course, in a moment this circle will be different and the radius too. So, for the sake of simplicity, we will first consider motion in a circle.

Angular speed $\vec{\omega}$ is a vector value equal to the first derivative of the turning angle with respect to time:

$$\vec{\omega} = \frac{d\vec{\varphi}}{dt}. \quad (3.1)$$

The angular speed in direction coincides with the angular displacement (along the axis of rotation according to the right-hand screw rule) (Fig. 3.2).

Angular velocity ω is measured in radians per second (rad/s).

Connection of linear and angular velocities. A material point moves in a circle radius R. Over time dt, her radius vector returned to the angle $d\varphi$, and she traveled the path $dS = R \cdot d\varphi$. (see Fig. 3.1.) Linear speed of the point $\upsilon = \frac{dS}{dt} = R\frac{d\varphi}{dt} = R\omega$.

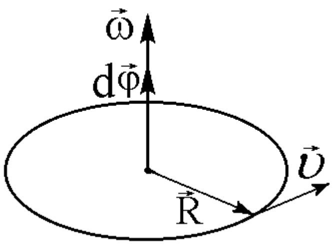


Fig. 3.2

In vector form

$$\vec{\upsilon} = \left[\vec{\omega} \cdot \vec{R}\right]. \quad (3.2)$$

Here, the square brackets mean a mathematical operation – the vector product of the vectors $\vec{\omega}$ and $\vec{R}$. The direction of the vector $\vec{\omega}$ in our case (Fig. 3.2) coincides with the direction of the axis of rotation "up".

In scalar form $\upsilon = \omega \cdot R \cdot \sin\left(\widehat{\vec{\omega} \cdot \vec{R}}\right)$. In our case $\omega \perp R$,

$$\upsilon = \omega \cdot R. \quad (3.3)$$

Each point of a rigid body in rotational motion has the same angular velocity regardless of the radius of the circle along which it moves, while the linear velocity depends on the radius of rotation.

Uniform movement in a circle is a movement in which the angles of rotation of the radius vector of a material point for any equal intervals of time are the same, i.e. $\omega = const$ equal to

$$\omega = \varphi/t. \quad (3.4)$$

The period of time during which the body makes a complete revolution while moving uniformly in a circle is called ***the period T:***

$$T = \frac{2\pi}{\omega}. \quad (3.5)$$

The rotation frequency ν shows how many revolutions in a circle the body makes per unit of time:

$$\nu = \frac{1}{T}. \quad (3.6)$$

Rotational frequency is measured in hertz (Hz).

Angular speed ω and frequency ν are related by the ratio:

$$\omega = 2\pi \cdot \nu \ . \qquad (5.7)$$

3. Angular acceleration

Angular acceleration characterizes the rate of change of angular velocity per unit of time. Angular acceleration $\vec{\beta}$ is a vector value equal to the first derivative of angular velocity with time:

$$\vec{\beta} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\varphi}}{dt^2} . \qquad (3.8)$$

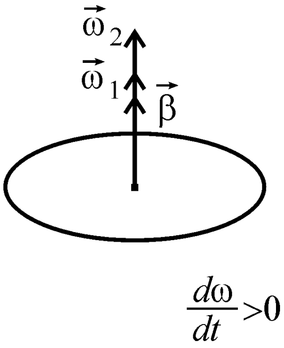


Fig. 3.3 a

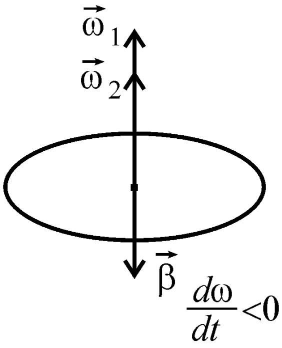


Fig. 3.3 б

The vector of angular acceleration $\vec{\beta}$ is directed along the axis of rotation in the direction of the vector of elementary increment of angular velocity $d\vec{\omega}$. If the movement is accelerated, the directions $\vec{\beta}$ and $\vec{\omega}$ coincide (Fig. 3.3 a); if the movement is slowed down, the directions $\vec{\beta}$ and $\vec{\omega}$ are also opposite (Fig. 3.3 b).

Angular acceleration β is measured in *radians per second squared* (rad/s^2).

Let's consider some cases of rotational motion:

1. $\beta = 0$ – ***uniform movement***.

Let's calculate the angular displacement φ: = const. $\varphi : \beta = \frac{d\omega}{dt} = 0 \Rightarrow \omega$ = const. By definition $\omega = \frac{d\varphi}{dt}$, whence $d\varphi = \omega \cdot dt$, and $\varphi = \int \omega \cdot dt = \omega t + \varphi_0$.

2. $\beta = const \neq 0$ – ***uniform motion***.

Angular velocity ω and angular displacement φ at any instant of time can be calculated, knowing the acceleration β:

By the definition of angular acceleration $\beta = \frac{d\omega}{dt}$, whence $d\omega = \beta \cdot dt$, a $\omega = \int \beta \cdot dt = \beta t + \omega_0$; by the definition of angular velocity $\omega = \frac{d\varphi}{dt}$, whence $d\varphi = \omega \cdot dt$, and $\varphi = \int \omega \cdot dt = \int (\omega_0 + \beta t) dt = \varphi_0 + \omega_0 t + \frac{\beta t^2}{2}$. So:

$$\omega = \omega_0 + \beta t , \qquad (3.9)$$

$$\varphi = \varphi_0 + \omega_0 t + \frac{\beta t^2}{2} . \qquad (3.10)$$

4. Relationship between normal and tangential acceleration with angular velocity and angular acceleration

The tangential component of acceleration , , thus

$$a_\tau = R\frac{d\omega}{dt} = R\beta \,. \qquad (3.11)$$

Normal component of acceleration or centripetal acceleration $a_n = \frac{\upsilon^2}{R}$. Let's substitute the value of linear speed into this formula $\upsilon = \omega R$. We will get:

$$a_n = \frac{\upsilon^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R \,. \qquad (3.12)$$

Full acceleration $a = \sqrt{a_\tau{}^2 + a_n{}^2}$. Substituting here the just obtained values of normal and tangential acceleration, we have

$$a = \sqrt{\omega^4 R^2 + \beta^2 R^2} = R\sqrt{\omega^4 + \beta^2} \,. \qquad (3.13)$$

5. Moment of inertia of a material point. The moment of inertia of a rigid body. Moments of inertia of some bodies of regular geometric shape. Steiner's theorem

The moment of inertia of a material point relative to the axis of rotation is called the product of the mass of this point by the square of the distance r to the axis of rotation.

$$J = mr^2 \,. \qquad (3.14)$$

So, this is a scalar quantity. The unit of moment of inertia is kilogram·meter squared (kg·m^2).

The moment of inertia of a solid body relative to a certain axis of rotation is called the sum of the products of the mass of each material particle of the body by the square of its distance to the axis of rotation:

$$J = \sum_n m_i r_i{}^2 \,, \qquad (3.15)$$

where r_i is the distance of the *i*-th point with mass m_i to the axis of rotation.

With continuous distribution of mass over the entire volume of the body:

$$J = \int_V r^2 dm \,. \qquad (3.16)$$

The integration is performed over the entire volume *V* of the body.

Calculating the integral (3.16) for bodies of different geometric shapes with a uniform mass distribution over the volume ($\rho = const$) gives the following formulas for determining their moments of inertia:

moment of inertia of a solid cylinder, disk relative to the central longitudinal axis

$$J = \frac{1}{2} mR^2 ,$$

where R is the radius of the cylinder (disk);

moment of inertia of a thin-walled cylinder (thin hoop) relative to the central longitudinal axis

$$J = mR^2 ,$$

where R is the radius of the cylinder;

the moment of inertia of a solid sphere relative to an axis passing through the center of the sphere

$$J = \frac{2}{5} mR^2 ,$$

where R – радіус кулі;
R – the radius of the sphere.

the moment of inertia of a thin rod with a length ℓ relative to an axis perpendicular to it passing through its middle

$$J = \frac{1}{12} m\ell^2 .$$

Using ***Steiner's theorem***, the moment of inertia of a body relative to any axis can be found if the moment of inertia of a body relative to a parallel axis passing through the center of mass is known. Steiner's theorem: the moment of inertia of a body relative to any axis of rotation J is equal to the sum of the moment of inertia of a body J_c relative to an axis parallel to it, passing through the center of mass of the body, and the product of the mass of the body by the square of the distance d between these axes (Fig. 3.4):

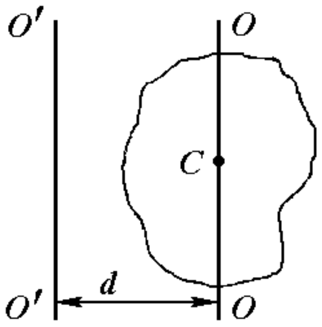


Fig. 3.4

$$J = J_c + md^2 / \qquad (3.17)$$

We use Steiner's theorem to determine the moment of inertia of a thin rod of mass m and length ℓ relative to an axis passing perpendicular to the rod through its $J_c = \frac{1}{12} m\ell^2$ end $d = \frac{\ell}{2}$. Taking into account that and , we get $J = \frac{m\ell^{2\,2}}{12} + m\frac{\ell^2}{4} = m\frac{\ell^2}{3}$.

Lecture 8
KINEMATICS AND DYNAMICS OF ROTATIONAL MOTION (CONTINUED)

6. The moment of strength. Moment of impulse

Consider the rotation of a body relative to an axis under the action of a force lying in a plane perpendicular to this axis. Let's draw a radius vector $\vec{r}$ from the axis to the point of force application in this plane (Fig. 3.5).

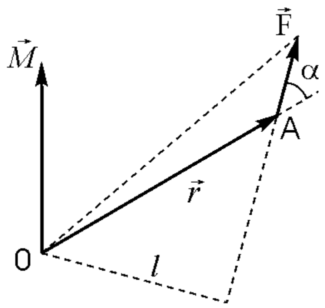


Fig. 3.5

The moment of a force $\vec{M}$ ***relative to a fixed point O*** is a physical quantity determined by the vector product of the radius-vector $\vec{r}$ of the point of application of the force and the force itself $\vec{F}$:

$$\vec{M} = \left[\vec{r} \cdot \vec{F}\right]. \tag{3.18}$$

The moment of force is an axial vector directed along the axis of rotation (perpendicular to the plane in which the vectors lie $\vec{r}, \vec{F}$) according to the right-hand helix rule.

The unit of moment of force is *newton meter* (N·m).

The modulus of the vector $\vec{M}$ is equal to

$$M = rF\sin\alpha = Fl \ , \tag{3.19}$$

where $l = r\sin\alpha$ is the shoulder of the force (*the shoulder* is the shortest distance from point O to the line of action of the force);

α is the angle between $\vec{r}$ and $\vec{F}$.

Therefore, the modulus of the moment of force is equal to the product of the magnitude of the force by the arm of the force.

Moment of force is also called rotational moment. The truly "rotating" capabilities of the force depend not only on its magnitude, but also on the shoulder.

The ***moment of force relative to an axis*** M_z is taken as the projection onto this axis of the moment of force relative to a point lying on this axis.

The moment of momentum of a material point relative to a stationary point O is called the value determined by the vector product of the radius-vector $\vec{r}$ of the point drawn from the point of rotation by the momentum of this material point (Fig. 3.6):

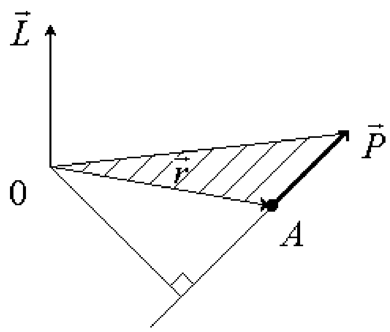


Fig. 3.6

$$\vec{L} = \left[\vec{r} \cdot \vec{p}\right] = \left[\vec{r} \cdot m\vec{\upsilon}\right]. \tag{3.20}$$

The momentum moment is an axial vector directed along the axis of rotation (perpendicular to the plane in which the vectors lie $\vec{r}, \vec{p}$) according to the right-hand helix rule.

In scalar form:

$$L = rp\sin\left(\widehat{\vec{r}\vec{p}}\right). \tag{3.21}$$

The moment of momentum is also called the moment of the amount of motion, the angular moment.

The unit of moment of momentum is *kilogram · meter squared per second* (kg·m^2/s).

The moment of momentum relative to a fixed axis Z is called a scalar value L_z, which is equal to the projection onto this axis of the vector of the moment of momentum determined relative to an arbitrary point *O* of this axis.

The moment of momentum of a solid body relative to an axis is equal to the sum of the moments of momentum of individual material points of this body relative to the same axis: $L_z = \sum_I L_{i_z}$ $= \sum_i r_i m_i \upsilon_i$. Using the relationship between the linear velocity and the angular velocity, which is the same for all points $\upsilon_i = \omega \cdot r_i$, we obtain:

$$L_z = \sum_i \omega \cdot m_i r_i^2 = J_z \omega, \quad (3.22)$$

where J_z is the moment of inertia of the rigid body relative to the given axis of rotation.

Considering that the directions $\vec{\omega}$ and $\vec{L}$ coincide, we have for a rigid body rotating relative to the axis

$$\vec{L} = J\vec{\omega}. \quad (3.23)$$

Let's compare this with the definition of body momentum, which is a dynamic characteristic of translational motion $\vec{p} = m\vec{\upsilon}$. We see that these equalities are quite similar in form. The first can be obtained from the second by a simple substitution: $\vec{p} \to \vec{L}$; $m \to J$; $\vec{\upsilon} \to \vec{\omega}$.

7. Kinetic energy of a rotating body

During the rotational movement of the body around a fixed axis, each material point by mass m_i moves in a circle of radius r_i with a linear speed υ_i. In general, all these values are different for different points $W_{k\,i} = \frac{m_i \upsilon_i^2}{2}$. However, all points have the same angular velocity ω. Let's use the formula for the kinetic energy of a material point and the formula for the relationship between linear velocity and angular velocity $\upsilon_i = \omega \cdot r_i$. The kinetic energy of a material point can be written as follows: $W_{k\,i} = \frac{m_i r_i^2 \omega^2}{2} = \frac{J_i \omega^2}{2}$ Here it is taken into account that $J_i = m_i r_i^2$.

The kinetic energy of a rotating body can be found as the sum of the kinetic energies of the material points that make up the body:

$$W_k = \sum_i W_{k_i} = \sum_i \frac{J_i \omega^2}{2} = \frac{J\omega^2}{2}, \quad (3.24)$$

where J is the moment of inertia of the body relative to the axis of rotation.

A comparison of formula (3.24) with the formula for the kinetic energy of a body moving forward $W_k = \frac{m\,\upsilon^2}{2}$ shows that the moment of inertia of rotational motion is a measure of the body's inertia.

If a body rolls (simultaneously moves forward and rotates), then its kinetic energy is equal to the sum of the kinetic energies of translational and rotational movements:

$$W_k = \frac{m\upsilon^2}{2} + \frac{J\omega^2}{2}, \tag{3.25}$$

where m is the mass of the rolling body;
υ – speed of the center of inertia (mass) of the body;
J – moment of inertia of the body relative to the axis passing through its center of mass;
ω is the angular velocity of the body.

8. The basic equation of the dynamics of rotary motion. Work with body rotation

The basic equation of the dynamics of rotational motion is equation 2 of Newton's law in relation to rotational motion. Let's find it for the movement of a material point of a solid body with a mass m around a circle of radius r under the action of a tangential force $F_\tau = ma_\tau = m\beta \cdot r$. The moment of this force relative to point *O* is determined by the formula $M = rF_\tau = rm\beta \cdot r = mr^2\beta$, i.e

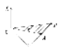
$$M = J\beta. \tag{3.26}$$

This equation for the rotational motion of a rigid body about a fixed axis coinciding with the principal axis of inertia passing through the center of mass has the form

$$\vec{M} = J\vec{\beta}, \tag{3.27}$$

If we consider the movement relative to the stationary *Z* axis, then the equation has the form

$$M_z = J_z\beta, \tag{3.28}$$

where  is the projection of the resulting moment of external forces on the *Z* axis;
J_z is the moment of inertia of the body relative to the *Z* axis.

Expressions (3.27) and (3.28) are ***equations of the dynamics of rotational motion of a rigid body.***

Let's pay attention to the similarity of equation (3.27) with equation 2 of Newton's law for translational motion $\vec{F} = m\vec{a}$. We can get the first by replacing the second $\vec{F} \to \vec{M}$; $m \to J$; $\vec{a} \to \vec{\beta}$.

Let's find an expression for ***elementary work dA during body rotation***. From the section "Work, power, energy" we know that the energy of a moving body increases by the amount of work expended, i.e. $dA = dW_k$. Given that $W_k = \frac{J_z\omega^2}{2}$, we get $dA = J_z\omega \cdot d\omega = J_z\omega dt\frac{d\omega}{dt}$; or taking into account that $\omega dt = d\varphi$, $\frac{d\omega}{dt} = \beta$ and equation (3.28),

$$dA = M_z d\varphi. \tag{3.29}$$

We will obtain an expression for the ***power during rotational motion***, taking into account that $N = \frac{dA}{dt}$ and expression (3.29), we will obtain:

$$N = M_Z\frac{d\varphi}{dt} = M_Z\omega. \tag{3.30}$$

9. Law of conservation of angular momentum

Let's get another expression of the equation of the dynamics of the rotational motion of a rigid body $\vec{L} = J\vec{\omega}$, namely, through the moment of momentum. We proceed from the definition of the moment of momentum of a rigid body. We differentiate this equation by time, considering the moment of inertia, $\frac{d\vec{L}}{dt} = J\frac{d\vec{\omega}}{dt} = J\vec{\beta} = \vec{M}$, where $\vec{M}$ is the total resulting moment of external forces, or the moment of the net force. We receive

$$\vec{M} = \frac{d\vec{L}}{dt}. \qquad (3.31)$$

We arrived at a more general form of ***the equation (law) of rotational motion***: the rate of change of the moment of momentum of the system relative to a fixed axis is equal to the resulting moment relative to the same axis of all external forces acting on the system.

It follows from the basic equation of the dynamics of rotational motion (3.31): if the moment $\vec{M}$ of external forces relative to the axis of rotation is zero, then

$$\frac{d\vec{L}}{dt} \equiv 0 \quad \text{and} \quad \vec{L} = \text{const}. \qquad (3.32)$$

In a closed system, the moment of external forces is $\vec{M} \equiv 0$. Expression (3.32) is called t***he law of conservation of angular momentum***: the angular momentum of a closed system of bodies is conserved, that is, it does not change over time.

The law of conservation of angular momentum is a fundamental law of nature. The law of conservation of angular momentum follows from the isotropy of space. For a body rotating around a fixed axis and in the absence of a moment of external forces relative to the same axis, the moment of momentum relative to this axis is also conserved. The law of conservation of angular momentum can be generalized to any open system of bodies: if the resultant moment of all external forces applied to the system relative to some fixed axis is identically equal to zero, then the angular momentum of the system relative to the same axis does not change with time: $M_z \equiv 0 \to L_z = const$. A closed system is a special case of this more general case.

An excellent demonstration of the law of conservation of angular momentum is Zhukovsky's chair, which is a rotating chair with a disc-shaped seat. During the demonstration of a person sitting on a bench with dumbbells clamped in his outstretched hands, the chair is rotated with an angular velocity ω_1 and given the opportunity to rotate itself. The man-bench system is closed (neglecting friction and air resistance). Therefore $L = J\omega$, the moment of momentum of the system relative to the axis of rotation $J_1\omega_1 = J_2\omega_2$ is preserved. Since , the product of the moment of inertia of the system and its angular velocity ($J_1\omega_1 = J_2\omega_2$) is preserved. If a person presses the dumbbells to himself, then the moment of inertia of the system will decrease (becomes J_2), and the angular velocity will increase (becomes ω_2).

The action of a gyroscope is based on the law of conservation of momentum - a massive homogeneous body that rotates with a high angular velocity around its axis of symmetry, which is free, that is, does not change its orientation in space. Driven into rotation and left to itself, the gyroscope preserves the orientation in space (as $\vec{L} = const$) of devices and devices connected with it (compasses, guns in a tank, autopilot systems in an airplane, etc.). The consequence of the preservation of the moment of momentum for a separate body moving in a central force field (that is, in a field whose forces depend only on the distance to the force center, as is the case with the movement of the planets around the Sun, satellites around the planets), is the preservation of the plane of rotation of the body (satellite, planet), as well as constancy of sectorial velocities of planets (Kepler's 2nd law).

10. Comparison of the mechanics of translational and rotational motion

At the end of the topic "Mechanics of rotational motion", we will give a comparative table of the values of the mechanics of translational and rotational movements, or otherwise, a table of analogies.

Table.

KINEMATICS	
PROGRESSIVE MOVEMENT	**ROTATIONAL MOVEMENT**
S **- path;** $d\vec{r}$ **– movement element;** $\vec{\upsilon}=\frac{d\vec{r}}{dt}$ **– speed;**	φ **– turning angle;** $d\vec{\varphi}$ **– element of angular movement;** $\vec{\omega}=\frac{d\vec{\varphi}}{dt}$ **– angular velocity;**
KINEMATICS	
PROGRESSIVE MOVEMENT	ROTATIONAL MOVEMENT
$\vec{a}=\frac{d\vec{\upsilon}}{dt}=\frac{d^2\vec{r}}{dt^2}$ – acceleration; $a=\frac{d\lvert\vec{\upsilon}\rvert}{dt}=\frac{d^2S}{dt^2}$ – acceleration module; $a_\tau=\frac{d\upsilon}{dt}$ – tangential component of acceleration; $a_n=\frac{\upsilon^2}{R}$ – normal component of acceleration;	$\vec{\beta}=\frac{d\vec{\omega}}{dt}=\frac{d^2\vec{\varphi}}{dt^2}$ – angular acceleration; $\beta=\frac{d\lvert\vec{\omega}\rvert}{dt}=\frac{d^2\varphi}{dt^2}$ – module of angular acceleration; $a_\tau=R\beta$ – tangential component of acceleration; $a_n=\omega^2R$ – normal component of acceleration;
$\vec{a}=\vec{a}_\tau+\vec{a}_n$ – full acceleration of a point in vector form; $a=\sqrt{a_\tau{}^2+a_n{}^2}$ – point acceleration module.	
Smooth movement	
$a=0$, $\vec{\upsilon}=\text{const}$, $S=\upsilon\cdot t+S_0$.	$\beta=0$, $\vec{\omega}=\text{const}$, $\varphi=\omega t+\varphi_0$.
Smooth movement	
$a=const$, $\upsilon=\upsilon_0+at$, $S=S_0+\upsilon_0t+\frac{at^2}{2}$	$\beta=const$, $\omega=\omega_0+\beta t$, $\varphi=\varphi_0+\omega_0t+\frac{\beta t^2}{2}$.
DYNAMICS	
PROGRESSIVE MOVEMENT	**ROTATIONAL MOVEMENT**
m – mass; $\vec{F}$ - strength; $\vec{p}=m\vec{\upsilon}$ – impulse; $A=\vec{F}\vec{S}$ - work; $N=\vec{F}\vec{\upsilon}$ - power	J – moment of inertia; $\vec{M}$ – moment of force; $\vec{L}=J\vec{\omega}$ – momentum moment; $A=\vec{M}\vec{\varphi}$ - work $N=\vec{F}\vec{\omega}$ - power
The basic law of dynamics	
$\vec{F}=m\vec{a}$; $\vec{F}=\frac{d\vec{p}}{dt}$.	$\vec{M}=J\vec{\beta}$; $M_Z=J\beta_Z$; $\vec{M}=\frac{d\vec{L}}{dt}$; $M_Z=\frac{dL_Z}{dt}$.
Kinetic energy	
$W_k=\frac{m\upsilon^2}{2}$.	$W_k=\frac{J\omega^2}{2}$.
Laws of conservation	
$\vec{p}=\text{const}$ **at** $\vec{p}=0$.	$\vec{L}=\text{const}$ **at** $\vec{M}=0$.

Lecture 9
Topic 4
KINEMATICS AND DYNAMICS OF OSCILLATORY MOVEMENT

Plan

1. Harmonic oscillations and their characteristics.
2. Speed, acceleration and energy of the body during harmonic oscillations.
3. Harmonic oscillator. Elastic, physical and mathematical pendulums.
4. Compilation of harmonic oscillations.
5. Attenuating oscillations.
6. Forced oscillations.
7. Wave processes. Transverse and longitudinal waves. Plane wave equation.
8. Acoustic waves. Sound and its characteristics.

1. Harmonic oscillations and their characteristics

Oscillations are movements or processes characterized by a certain repeatability in time. An example of oscillations: the movement of a clock pendulum, a change in the current strength in an electrical network, light processes. By their nature, oscillations are divided into mechanical and electromagnetic. Oscillations of different nature (mechanical, electromagnetic) are described by the same characteristics and equations. It is only necessary to determine the physical quantity involved in the oscillations.

Oscillations are called free (or natural) if they occur at the expense of the initially provided energy in the absence of external influences on the oscillating system. Such oscillations are oscillations with a constant amplitude and frequency. The frequency of free oscillations is called the natural frequency of the oscillating system. An example can be a long pendulum, deflected to a small angle; it can oscillate for a long time without decreasing in amplitude.

However, the presence of friction forces in real conditions leads to damping of oscillations. In order to obtain undamped oscillations in a real oscillating system, it is necessary to compensate for energy losses.

The simplest type of oscillations are ***harmonic oscillations*** - oscillations in which the oscillating S quantity changes according to the cosine (sine) law:

$$S = A\cos(\omega_0 t + \varphi_0), \quad (4.1a)$$

$$S = A\sin(\omega_0 t + \varphi_0), \quad (4.1б)$$

where A is the amplitude of oscillations. ***The amplitude*** of oscillations is called the module of the largest displacement of a point from the equilibrium position;

$(\omega_0 t + \varphi_0)$ – ***phase of oscillations*** – the value under the cosine sign;

ω_0 – ***circular*** *or* ***cyclic frequency of oscillations***;

φ_0 is ***the initial phase of oscillations***, that is, the phase at time $t = 0$.

In oscillating motion, the value S takes on values from $-A$ to $+A$.

The time t dependence graph S is a cosine (Fig. 7.1, the initial phase is zero).

The oscillation period T is the minimum period of time during which the movement is completely repeated, the oscillation phase receives an increment of 2π:

$$\omega_0(t + T) + \varphi_0 = (\omega_0 t + \varphi_0) + 2\pi ,$$

where

$$T = \frac{2\pi}{\omega_0}. \quad (4.2)$$

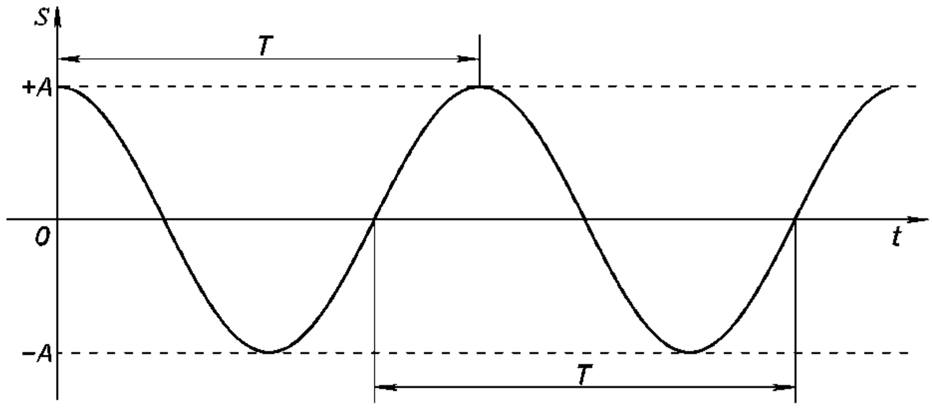


Fig. 4.1

The quantity ν, the inverse of the oscillation period, which is equal to the number of oscillations per unit of time, is ***called the oscillation frequency***,

$$\nu = \frac{1}{T}. \quad (4.3)$$

The unit of frequency is *hertz* (Hz).

Comparing (4.2) and (4.3), we obtain

$$\omega_0 = 2\pi\nu. \quad (4.4)$$

It can be seen from (4.1) that the first and second time derivatives S of the harmonically oscillating quantity also carry out harmonic oscillations at the same frequency

$$\frac{dS}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi_0), \quad (4.5a)$$

$$\frac{d^2S}{dt^2} = -A\omega_0{}^2 \cos(\omega_0 t + \varphi_0) = -\omega_0^2 S. \quad (4.5б)$$

It follows from equation (4.5b) that the quantity S, which oscillates harmonically, satisfies the differential equation

$$\frac{d^2S}{dt^2} + \omega_0{}^2 S = 0. \quad (4.6)$$

This is ***the equation of harmonic oscillations in differential form***. Its solution is equation (4.1a) or (4.1b).

2. Speed, acceleration and energy of the body during harmonic oscillations

Examples of mechanical harmonic oscillatory movements are small oscillations of spring, mathematical and physical pendulums.

A material point carries out harmonic oscillations along the coordinate x axis near the equilibrium position taken as the coordinate origin. The point coordinate value changes over time according to the cosine law:

$$x = A\cos(\omega_0 t + \varphi_0). \qquad (4.7)$$

According to the definition, the speed υ and acceleration a of a point are equal, respectively

$$\upsilon = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t + \varphi_0), \qquad (4.8)$$

$$a = \frac{d^2x}{dt^2} = \frac{d\upsilon}{dt} = -A\omega_0{}^2 \cos(\omega_0 t + \varphi_0),$$

taking into account (4.7)

$$a = -\omega_0{}^2 x. \qquad (4.9)$$

The force $F = ma$ acting on a material point by an oscillating mass m,

$$F = -m\omega_0{}^2 x. \qquad (4.10)$$

The force is proportional to the displacement of the material point from the equilibrium position and is directed in the opposite direction (toward the equilibrium position).

The kinetic energy of a harmonically oscillating material point,

$$W_k = \frac{m\upsilon^2}{2} = \frac{mA^2\omega_0{}^2}{2}\sin^2(\omega_0 t + \varphi_0). \qquad (4.11)$$

The potential energy of a material point that oscillates harmonically under the action of an elastic force F,

$$W_n = -\int_0^x F dx = \frac{m\omega_o{}^2 x^2}{2} = \frac{mA^2\omega_0{}^2}{2}\cos^2(\omega_0 t + \varphi_0). \qquad (4.12)$$

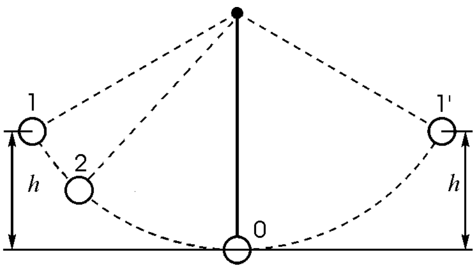


Fig. 4.2

Adding (4.11) and (4.12), we get the total energy formula:

$$W = W_k + W_n = \frac{mA^2\omega_0{}^2}{2}. \qquad (4.13)$$

Conclusion: during oscillatory motion, kinetic energy is transformed into potential energy and vice versa, at any point between the positions of equilibrium and maximum deviation, the body has both kinetic and potential energy, but their sum, that is, the total mechanical energy of the system, is constant and is determined by expression (4.13).

The transformation of energy during harmonic oscillations is easy to observe using the example of a mathematical pendulum (Fig. 4.2). At points 1 and 1', the potential energy of the mathematical pendulum is maximum, and the kinetic energy is zero. At some point 2, the kinetic energy is equal to the potential energy. At point 0, the kinetic energy is maximum, and the potential energy is zero.

3. Harmonic oscillator. Elastic, physical and mathematical pendulums

A harmonic oscillator is a model used in solving linear problems of classical and quantum physics. Spring, physical, mathematical pendulums, oscillating circuit are examples of a harmonic oscillator. A harmonic oscillator carries out oscillations that can be described by an equation of the form $\ddot{S} + \omega_0^2 S = 0$.

A spring pendulum (Fig. 4.3) is a system that consists of a weight with a mass m attached to a spring and oscillates along a certain direction under the action of the elastic force $F = -kx$, where k is the stiffness of the spring. The equation of motion of the pendulum is $m\ddot{x} = -kx$, or $\ddot{x} + \frac{k}{m}x = 0$, where $\ddot{x} = \frac{d^2x}{dt^2}$. From expressions (4.6) and (4.7) we can conclude that the spring pendulum carries out harmonic oscillations according to the law $x = A\cos(\omega_0 t + \varphi_0)$ with a cyclic frequency

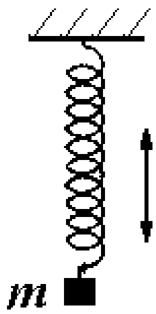


Fig. 4.3

$$\omega_0 = \sqrt{k/m} \qquad \textbf{(4.14)}$$

and period

$$T = 2\pi\sqrt{m/k}\,. \qquad (4.15)$$

The potential energy of the spring pendulum is equal to $W_n = kx^2/2$.

A physical pendulum (Fig. 4.4) is a completely solid body that, under the influence of gravity, oscillates around a fixed axis that does not pass through its center of inertia (weight). Let's deviate the pendulum from the equilibrium position by a small angle α ($\sin\alpha \approx \alpha$). According to the main equation of the dynamics of rotary motion, the moment of force M, which returns the body to the equilibrium position, is equal to $M = J\beta = J\ddot{\alpha} = F_\tau d = -mgd\sin\alpha = -mgd\alpha$, where J is the moment of inertia of the physical pendulum; d is the distance between the suspension point of the pendulum and the center of inertia (weight), $\ddot{\alpha} = \frac{d^2\alpha}{dt^2}$. The sign "–" is due to the fact that the directions F_τ and α are always opposite. The equation of motion can be written in the form $\ddot{\alpha} + \frac{mgd}{J}\alpha = 0$. Taking into account the fact that $\omega_0 = \sqrt{\frac{mgd}{J}}$, we will obtain the equations of motion of the physical pendulum in the differential form:

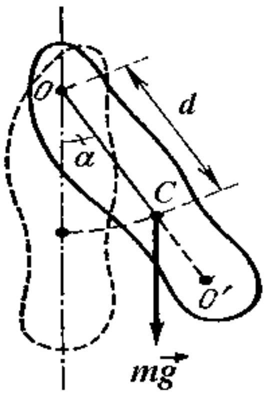


Fig. 4.4

$$\ddot{\alpha} + \omega_0^2\alpha = 0. \qquad (4.16)$$

Oscillation period of the physical pendulum

$$T = 2\pi\sqrt{J / mgd}\ . \qquad (4.17)$$

A mathematical pendulum (Fig. 4.5) is a system that consists of a material point with a mass m suspended on an inextensible weightless thread and oscillates under the influence of gravity. The moment of inertia of a mathematical pendulum $J = m\ell^2$, ℓ is the length of the pendulum. Since the mathematical pendulum is a case of a physical pendulum (all the mass is concentrated in one point - the center of inertia), then we substitute the formula for the moment of inertia of the mathematical pendulum into expression (4.17) and obtain the period of oscillations of the mathematical pendulum

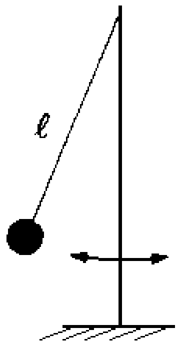


Fig. 4.5

$$T = 2\pi\sqrt{\ell / g}\ . \qquad (4.18)$$

Comparing formulas (4.17) and (4.18), it can be seen that the oscillation period of the physical pendulum coincides with the oscillation period of the mathematical pendulum, length

$$L = \frac{J}{md}\ , \qquad (4.19)$$

which is called ***the reduced length of the physical pendulum***. From (4.17) and (4.19) we obtain the following expression for the oscillation period of a physical pendulum:

$$T = 2\pi\sqrt{L / g}\ . \qquad (4.20)$$

4. Compilation of harmonic oscillations

Addition of two harmonic oscillations occurring along the same direction. A mass point m simultaneously participates in two harmonic oscillations directed along the same straight line. It is necessary to find the resulting oscillation. Let's add harmonic oscillations of the same direction and the same frequency ω_0

$$x_1 = A_1\cos(\omega_0 t + \varphi_{01}),$$

$$x_2 = A_2\cos(\omega_0 t + \varphi_{02}).$$

The equation of the resulting oscillation has the form

$$x = x_1 + x_2 = A\cos(\omega_0 t + \varphi_0). \qquad (4.21)$$

The resulting oscillation occurs with the amplitude A, which is found by the method of vector diagrams and is equal to the modulus of the sum of the vectors of the component amplitudes $\vec{A}_1$ and $\vec{A}_2$:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_{02} - \varphi_{01})}\ . \qquad (4.22)$$

The initial phase φ_0 is determined from the ratio

$$tg\varphi_0 = \frac{A_1 \sin\varphi_{01} + A_2 \sin\varphi_{02}}{A_1 \cos\varphi_{01} + A_2 \cos\varphi_{02}}. \quad (4.23)$$

Thus, if the body participates in two harmonic oscillations of the same direction and the same frequency ω_0, then it also performs harmonic oscillations in the same direction and with the same frequency ω_0, as the added oscillations. The amplitude of the resulting oscillations depends on the added phase $(\varphi_{02} - \varphi_{01})$ difference. If $\varphi_{02} - \varphi_{01} = \pm 2m\pi$ $(m = 0,1,2,...)$, then $A = A_1 + A_2$; if $\varphi_{02} - \varphi_{01} = \pm(2m+1)\pi$ $(m = 0,1,2,...)$, then $A = |A_1 - A_2|$.

Addition of mutually perpendicular oscillations. Consider the result of the addition of two harmonic oscillations of the same frequency ω_0 occurring in mutually perpendicular directions along the x and y axes. For simplicity, we will choose the beginning of the countdown so that the initial phase of the first oscillation is equal to zero: $x = A\cos\omega_0 t$; $y = B\cos(\omega_0 t + \varphi)$.

The phase difference of both oscillations is equal to φ, and *A* and *B* are the amplitudes of the added oscillations.

The equations of motion of the resulting oscillations can be found by excluding time from these equations,

$$\frac{x^2}{A^2} - \frac{2xy}{AB}\cos\varphi + \frac{y^2}{B^2} = \sin^2\varphi. \quad (4.24)$$

This is the equation of an ellipse. The orientation of the axes of the ellipse and its dimensions depend on the amplitude of the added oscillations and the phase difference φ. If $\varphi = m\pi$ ($m = 0, \pm1, \pm2...$), then the ellipse degenerates into a line $y = \pm(B/A)x$ segment that makes an angle $\varphi = arctg\left(\frac{B}{A}\cos m\pi\right)$ with the x axis. If $\varphi = (2m+1)\frac{\pi}{2}$ **(**$m = 0, \pm1, \pm2...$**)**, then equation (4.24) has the form $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$. This is the equation of an ellipse, the axes of which coincide with the coordinate axes, and its semi-axes are equal to the corresponding amplitudes. In addition, if $A = B$, then the ellipse degenerates into a circle. We come to a more complex trajectory in the case of a combination of oscillations in mutually perpendicular directions with different natural frequencies and initial phases. Such trajectories are called Lissajous figures.

Lecture 10
KINEMATICS AND DYNAMICS OF OSCILLATORY MOTION (CONTINUED)

5. Attenuating oscillations

The presence of friction force in real conditions leads to damping of oscillations. Oscillations with an amplitude that decreases with time are called decaying.

Consider the oscillations of a spring pendulum with mass m. It is stretched and released. The movement of the pendulum is affected by the resistance of the medium in which it oscillates. To overcome this resistance, energy is spent, and the oscillations of the pendulum gradually decrease, that is, the amplitude of the oscillations decreases (Fig. 4.6).

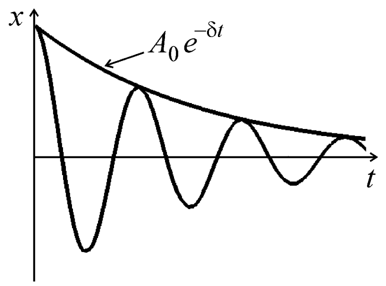


Fig. 4.6

Damping oscillations occur under the action of two forces $F=-kx$: elastic force; the resistance of the medium $F=-rv=-r\dot{x}$, proportional to the speed υ of the load, where $\upsilon=\dot{x}=\dfrac{dx}{dt}$.

According to Newton's second law, the equation of motion of the pendulum has the form

$$ma=-kx-r\upsilon, \quad \text{or} \quad m\ddot{x}=-kx-r\dot{x}, \tag{4.25}$$

where k is the stiffness coefficient of the spring;
r – medium resistance coefficient;
x – displacement of the load relative to the equilibrium position;
m – weight of the load;
a – acceleration of the load, $a=\ddot{x}=\dfrac{d^2x}{dt^2}$.

Equation (4.25) of the pendulum motion is written in the form

$$m\ddot{x}+r\dot{x}+kx=0. \tag{4.26}$$

If we divide equation (4.26) by m and introduce the notation

$$\omega_0{}^2=\frac{k}{m}, \qquad 2\delta=\frac{r}{m},$$

then we get ***the differential equation of damped oscillations of the pendulum***:

$$\ddot{x}+2\delta\,\dot{x}+\omega_0{}^2x=0, \tag{4.27}$$

where δ is ***the attenuation coefficient***.

It follows from the expression (4.27) that the pendulum oscillates according to the law

$$x=A_0e^{-\delta t}\cos(\omega t+\varphi_0), \tag{4.28}$$

where A_0 is the initial amplitude; $\omega=\sqrt{\omega_0{}^2-\delta^2}$ – cyclic frequency of damped oscillations of the system;

ω_0 is the natural cyclic frequency of free undamped oscillations of this pendulum at $\delta = 0$.

Amplitude A of damped oscillations

$$A = A_0 e^{-\delta t}. \quad (4.29)$$

Damping breaks the periodicity of oscillations. But, if the attenuation is small $\left(\delta^2 << \omega_0{}^2\right)$, you can use the concept of period. A period of damped oscillations

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0{}^2 - \delta^2}}. \quad (4.30)$$

The time τ during which the amplitude of damping oscillations will decrease e by a factor is called ***the relaxation time*** $\tau = 1/\delta$.

Over time τ, the system will perform N_e oscillations $N_e = \tau / T$.

For the quantitative characteristic of the rate of decrease of the amplitude of damping oscillations, the concept of damping decrement λ and the concept of logarithmic damping decrement are used θ.

The attenuation decrement is the ratio of amplitudes corresponding to moments of time that differ by a period:

$$\lambda = \frac{A(t)}{A(t+T)} = e^{\delta T}, \quad (4.31)$$

and its logarithm

$$\theta = \ell n \frac{A(t)}{A(t+T)} = \delta T = \frac{T}{\tau} \quad (4.32)$$

is called ***the logarithmic decay decrement***.

To characterize the oscillating system, the concept of factor is used. The quality of the oscillating system Q is proportional to the ratio of the energy $W(t)$ of the system to the reduction of this energy during the period of damped oscillations and is determined by the formula

$$Q = 2\pi \frac{W(t)}{W(t) - W(t+T)}. \quad (4.33)$$

Since the energy $W(t)$ is proportional to the square of the amplitude of oscillations $A(t)$, then

$$Q = 2\pi \frac{A^2(t)}{A^2(t) - A^2(t+T)} . \quad (4.34)$$

At small values of the logarithmic decrement of damping $(\theta << 1)$, the Q-factor of the oscillating system

$$Q = \frac{\pi}{\theta} = \frac{\omega_0}{2\delta} = \frac{1}{r}\sqrt{km}. \quad (4.35)$$

6. Forced oscillations

In order to obtain undamped oscillations in a real oscillating system, it is necessary to compensate for energy losses. This is possible if the oscillating body is acted upon by an external factor $X = X_0 \cos\omega t$ that changes periodically.

If we consider mechanical oscillations, an external $X(t)$ force plays a role. Oscillations that occur under the influence of an external force that changes periodically are called ***forced***. If the coercive force changes according to the harmonic law $F = F_0 \cos\omega t$, then the equation of oscillations of the spring pendulum in differential form is written as follows:

$$m\ddot{x} = -kx - r\dot{x} + F_0 \cos\omega t \,. \tag{4.36}$$

If we divide equation (4.36) by and enter the notation $\omega_0{}^2 = \frac{k}{m}$, $2\delta = \frac{r}{m}$, $f_0 = F_0 / m$, then we get the ***differential equation of the forced oscillations of the pendulum***

$$\ddot{x} + 2\delta\,\dot{x} + \omega_0{}^2 x = f_0 \cos\omega t \,, \tag{4.37}$$

Some time after the beginning of the periodic force, oscillations with the frequency of the external force are established. These oscillations are harmonic:

$$x = A\cos(\omega t + \varphi_0), \quad (t >> T) \,. \tag{4.38}$$

The amplitude of oscillations depends on the ratio of the natural frequency of the oscillating system and the frequency of the forcing force, as well as on the damping coefficient

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2) + 4\delta^2\omega^2}} \,. \tag{4.39}$$

The phenomenon of a sharp increase in the amplitude of forced oscillations when the frequency of the forcing force approaches the resonant frequency of the oscillating $\omega_{рез}$ system is called resonance.

Resonance frequency $\omega_{рез} = \sqrt{\omega_0^2 - 2\delta^2}$.

Resonance amplitude of mechanical vibrations $A_{рез} = \frac{f_0}{\sqrt{\omega_0^2 - 2\delta^2}}$.

The phenomenon of resonance is widely used in radio engineering, applied acoustics, in various vibrators and vibration stands. However, when designing machines and structures subjected to loads, in order to avoid their destruction, the possibility of harmful effects of resonance is taken into account.

7. Wave processes. Transverse and longitudinal waves. Plane wave equation

When considering mechanical oscillations, we were not interested in the processes that take place in the environment surrounding the oscillating system. We consider the environment to be continuous. The process of propagation of vibrations in a continuous medium is called ***a wave process (wave)***. Elastic waves on the liquid surface and electromagnetic waves are important and the most common. Special cases of elastic waves are sound and seismic waves, and electromagnetic waves are radio waves, light, and X-rays.

Oscillations excited at some point of the medium propagate in it with a finite phase speed υ. When a wave propagates, the particles of the medium do not move with the wave, but oscillate near their equilibrium positions. The basic property of all waves, regardless of their nature, is that the wave carries energy without transporting matter.

In the propagation of vibrations in elastic media, deformations of bodies and elastic forces arising from these deformations play a significant role. An example of such oscillations is the oscillation of an elastic rod or a stretched string. If one end of an elastic rod is given an oscillating motion, then this motion spreads along the entire rod. Such movements belong to the class of wave movements. In ***longitudinal waves***, the direction of wave propagation coincides with the direction of vibrations of the particles of the medium. An example is sound waves in gases and liquids. Another type of wave is ***transverse***. In them, particles of the medium oscillate in a direction perpendicular to the direction of wave propagation. When a wave propagates along a string, the points of the string are displaced perpendicular to it.

Only longitudinal waves occur inside liquids and gases, and both longitudinal and transverse waves occur in solids.

The concept of a harmonic wave is of particular importance in wave theory. ***An elastic wave*** is called ***harmonic*** if the vibrations of the particles of the medium corresponding to it are harmonic.

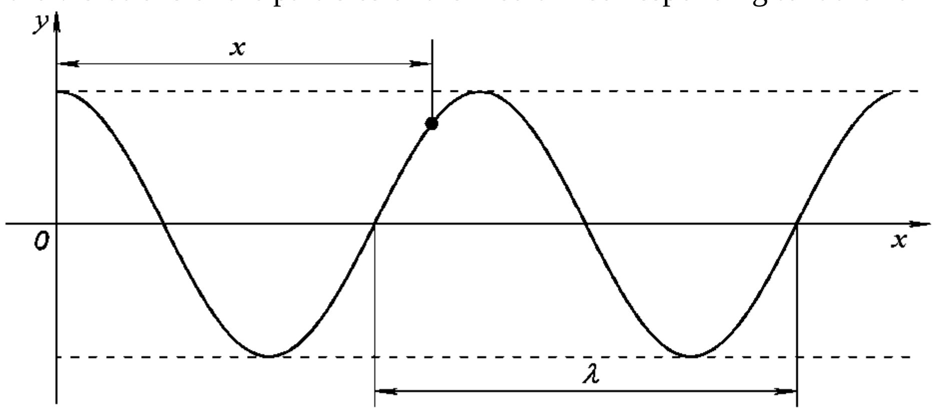


Fig. 4.7

In fig. 4.7 shows a harmonic wave propagating along the *x*-axis. The graph of the wave gives the dependence of the displacement of the particles of the medium on the distance to the source of oscillations at a given moment in time.

The distance between the nearest particles oscillating in the same phase is called the ***wavelength*** λ. The wavelength is equal to the distance that the wave travels in a period, i.e

$$\lambda = \upsilon \cdot T \; . \tag{4.40}$$

Taking into account that $T = 1/\nu$, where ν is the frequency of oscillations,

$$\upsilon = \lambda \cdot \nu \; . \tag{4.41}$$

The wave front is the geometric location of the points reached by the oscillation at a certain time.

A wave surface is a geometric locus of points that are in the same phase. If this surface is flat, the wave is flat, if it is spherical, the wave is spherical.

When undamped oscillations propagate along some direction called ***a beam***, the displacement of a particle of the medium lying on the beam is given by the equation

$$y = A\cos\left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda} + \varphi_0\right), \tag{4.42}$$

where A is the oscillation amplitude;
λ - wavelength;
$\omega = \frac{2\pi}{T}$ – circular frequency of oscillations;
x is the distance from the particle to the source of oscillations;
φ_0 – initial phase;
$\varphi = \left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda} + \varphi_0\right)$ - oscillation phase.

The wave number k determines the number of waves superimposed on a segment of length 2π m:

$$k = \frac{2\pi}{\lambda}. \tag{4.43}$$

The plane wave equation (4.42) can be rewritten in the form

$$y = A\cos(\omega t - kx + \varphi_0), \tag{4.44}$$

The value of the particle υ velocity is defined as the first derivative of the displacement with time:

$$\upsilon = \frac{dy}{dt} == -\frac{2\pi A}{T}\sin\left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda} + \varphi_0\right).$$

The value of the acceleration of a particle is defined as the first derivative of the velocity with time:

$$a = \frac{d\upsilon}{dt} = -\frac{4\pi^2 A}{T^2}\cos\left(\frac{2\pi}{T}t - \frac{2\pi x}{\lambda} + \varphi_0\right).$$

8. Acoustic waves. Sound and its characteristics

Sound is mechanical vibrations that propagate in an elastic medium with frequencies from 16 to 20,000 Hz, which are perceived by a special sense organ of humans and animals. The study of sound waves is considered in the branch of physics called acoustics. The propagation of sound waves in the environment is characterized by their speed. The speed of sound propagation depends on the elastic properties of the medium in which sound vibrations occur, its density, and temperature. We will give examples of the speed of sound in a gas, liquid, and solid body at room temperature: air – υ = 332 m/s; water – υ = 1450 m/s; iron – υ = 4900 m/s.

The intensity (or power) of sound is the value determined by the amount of sound energy passing through the surface of a unit area per unit time in the direction perpendicular to this surface:

$$I = \frac{W}{St}, \tag{4.45}$$

where I is sound power;
W – sound wave energy;
S – area;
t is time.

The unit of measurement of intensity is watt per *square meter* (W/m^2).

Loudness is a subjective characteristic of sound, related to its intensity and dependent on the frequency of oscillations. The human ear has the greatest sensitivity in the frequency range of 1-5 kHz. It was established that the volume of the sound increases with increasing intensity according to the logarithmic law. On this basis, we will introduce a characteristic - the level of sound intensity L:

$$L = \ell g \frac{I}{I_0}, \tag{4.46}$$

where I is the intensity of this sound;
I_0 is the intensity corresponding to the hearing threshold at a frequency of approximately 1000 Hz. ($I_0 = 10^{-12}$ W/m^2 for all sounds).

The unit of intensity level is *bel* (B), but a unit 10 times smaller is often used - *decibel* (dB). For example, the rustling of tree leaves is estimated at 10 dB, street noise at 70 dB. The physiological characteristic of sound is the *volume level* measured in the *background (background).* The volume level for a sound with a frequency of 1 kHz is equal to 1 background if its intensity level is equal to 1 dB.

The pitch of the tone (sound) depends on the frequency of the sound waves. As the frequency increases, the sound pitch increases, the sound becomes "higher", the sounds of "low" tones are low-frequency oscillations in the sound wave. There are special sound sources that emit almost a single frequency ("pure tone"). These are tuning forks.

Acoustic, sound resonance is a special case of mechanical resonance. A sounding body can carry out both free and forced oscillations under the action of an external periodic force. If the oscillation frequency of the external force coincides with its own oscillation frequency, resonance occurs. Consider two identical tuning forks. If we hit the stem of one tuning fork, then, it turns out, the other tuning fork will soon begin to sound. The sound wave from the first tuning fork creates a periodic force acting on the second tuning fork. The natural frequencies of the tuning forks are the same, and the amplitude of oscillations of the second tuning fork due to resonance turns out to be quite large. If you take tuning forks with different natural frequencies, then the second tuning fork will not sound.

In a closed room, multiple sound reflections occur from the walls, ceiling, floor and other objects. The human ear retains the sense of perceived sound for 0.1s. If the reflected sounds reach the human ear with shorter time intervals, then they are not perceived as separate sounds, but only amplify and continue the main sound. If the time interval between the moments when the main and reflected sound is heard exceeds 0.1s, then the reflected sounds are perceived separately as an ***echo***.

The frequency range from 16 to 20,000 Hz is called ***the sound range***. Inaudible mechanical vibrations with frequencies below 16 Hz are called ***infrasound***, and with frequencies above the sound range, that is, more than 20,000 Hz, are called ***ultrasound***.

An example of infrasound is the so-called "voice of the sea". The rarefaction and compression of a sea wave are transmitted to the space above the surface of the sea and generate an infrasonic wave. Ultrasound is widely used in technology, for example, to measure depth, underwater location (sonar), in such a field of science as ultrasonic flaw detection, in the pharmaceutical and food industry, in construction (determining the quality of concrete structures), in medicine (diagnosis, treatment, surgery). Many modern industrial technologies lead to the release of combustion products dangerous to human health into the air: dust, smoke, compounds of heavy metals. Ultrasonic vibrations are able to combine small particles of harmful substances into large, easily settling particles (coagulation process). Ultrasonic methods of water disinfection and disinfection are now widely used.

An important factor in the impact on the environment is the acoustic impact of industrial facilities - mechanical noise (noise from gearboxes, bearings, generators) and aerodynamic noise (arising from the rotation of impellers, turbines), which can be both in the range of audible sounds and in the range of infra- and ultrasound, harmful to human health. The normal level of sound intensity does not exceed 50-60 dB. Noise, the intensity level of which reaches 130 dB, is felt by the skin and causes pain.

Intensity levels of some sounds

SOUNDS	L, dB
Whisper	20
Silent conversation	40
Normal conversation	50
Cry	80
Motorcycle noise	100
Jet engine noise (at a distance of 5 m)	120
Pain threshold	130

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