# TO THE QUESTION OF THE MOVEMENT OF A QUADRATIC NONLINEAR OSCILLATOR WITH DRY FRICTION

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The purpose of research. The aim of the research is to derive and test formulas for calculating the displacement of the oscillator during motion and to determine the durations of oscillation half-cycles under dry friction conditions. It is possible to derive an exact recurrent relation for calculating the spans of damped free oscillations under the action of dry friction without constructing a solution of the differential equation of motion, if we use the energy method. But determining the oscillator's displacements over time requires solving the differential equation of motion.

Basic research materials.

The motion of the oscillator is described by the differential equation:

$$m\ddot{x} + c_1 x + c_2 \left| x \right| \cdot x + F_{fr} \operatorname{sign}\left( \dot{x} \right) = 0, \qquad (1)$$

with initial conditions:

$$x\left(0\right) = -a_0; \;\; \dot{x}\left(0\right) = 0, \qquad (2)$$

where $m$ – oscillator mass; $c_1$, $c_2$ – coefficients of elasticity; $F_{fr}$ – dry friction force; $a_0$ – initial deviation from equilibrium.

We will find solutions for hard and soft characteristics.

In the case of rigid power characteristics:

$$m\ddot{x} + c_1 x - c_2 x^2 + F_{fr} = 0, \qquad (3)$$

Suppose that $c_1 > 0$, $c_2 > 0$ and consider the first quarter of the cycle, where $x \in \left[ -a_0; 0 \right]$. The movement of the oscillator is described by the expression:

$$x(t) = b_2^2 \,\mathrm{sn}^2\left(\tau; \frac{b_2}{b_1}\right) - \mathrm{a}_0, \tag{4}$$

where $\mathrm{sn}\left(\tau; \dfrac{b_2}{b_1}\right)$ – elliptical sine of Jacobi, $b_1^2 = \dfrac{B}{2} + \sqrt{\left(\dfrac{B}{2}\right)^2 - A + a_0}$; $b_2^2 = \dfrac{B}{2} - \sqrt{\left(\dfrac{B}{2}\right)^2 - A + a_0}$, $A = \dfrac{3}{2c_2}\left(c_1 a_0 + \dfrac{2}{3} c_1 a_0^2 - 2F_{fr}\right)$; $B = \dfrac{3}{2c_2}\left(c_1 + \dfrac{2}{3} c_1 a_0\right)$.

Consider the second quarter of the cycle, where $x \in [0; a_1]$. In this area of movement $x \geq 0$, $\dot{x} \geq 0$ and equation (1) has the form:

$$my\frac{dy}{dx} + c_1 x + c_2 x^2 + F_{fr} = 0. \tag{5}$$

The formula for determining the movements of the oscillator in the second quarter using elliptical functions in the final form has the form:

$$x(t) = a_1 - \rho \frac{\mathrm{sn}^2(\xi; r)}{\mathrm{cn}^2(\xi; r)}, \tag{6}$$

where $\xi = \mathrm{F}\left(2\,\mathrm{arctg}\dfrac{\sqrt{a_1 - x}}{\sqrt{\rho}}; r\right)$, $\rho = \sqrt{a_1^2 + Da_1 + c}$; $d = \dfrac{D}{2}$; $r = \dfrac{\sqrt{\rho + d + a_1}}{\sqrt{2\rho}}$, $D = \dfrac{3}{2c_2}\left(c_1 + \dfrac{2}{3} c_2 a_1\right)$.

We use a similar approach for the variant of soft force characteristic, and we obtain the equation of motion in the form of dependences expressed in terms of elliptic Jacobi functions.

Conclusions.

The differential equation of free oscillations of an oscillator with a quadratically nonlinear force characteristic and dry friction has exact analytical solutions, which are expressed in terms of elliptic Jacobi functions. An additional numerical integration of the equation of motion on a computer confirmed the probability of the derived analytical formulas for calculating the oscillator displacements over time.